Quiet Sun 20 MHz Antenna Temperature Analysis

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Solar radio bursts are easy to observe with practically any receiver. The question arises: can we see the *quiet* Sun with a Radio Jove radio telescope? The Radio Jove telescope consists of a dual dipole phased array having a gain of about 8 dBi feeding a direct conversion receiver at 20.1 MHz with a bandwidth of around 6 kHz and an integration time of 100 ms.

Theory

The gain of an antenna is given by:[1]

$$G = \frac{4\pi}{\Omega_A} \qquad \text{(Eqn 1)}$$

where

G = Antenna gain as a power ratio

Ω_A = Antenna beam size, solid angle in steradians

Observed antenna temperature is given by: [2]

$$T_{ANT} = \frac{1}{\Omega_A} \iint T_S(\theta,\phi)\, P_N(\theta,\phi)\, d\Omega \qquad \text{(Eqn 2)}$$

where

$T_S(\theta,\phi)$ = Source temperature distribution

$P_N(\theta,\phi)$ = Normalized antenna pattern

$\Omega_A = \dfrac{4\pi}{G}$ Antenna beam size, solid angle in steradians

G = Antenna gain as a power ratio

For a source that is very small compared to the antenna's half power beam width (HPBW), and with the source located at the beam centerline, we may reasonably say that $P_N(\theta,\phi) \approx 1$ over the extent of the source within the beam.

Furthermore, the source will extend over some solid angle Ω_S and will have an average temperature over this solid angle. Thus, the source temperature distribution $T_S(\theta,\phi)$ may be represented by its average temperature T_{SRC}.

Therefore, for a small source ($\Omega_S \ll \Omega_A$), Equation 2 simplifies to:

$$T_{ANT} = \frac{\Omega_S}{\Omega_A} T_{SRC} \qquad\qquad \text{(Eqn 3)}$$

where

T_{SRC} = Source average temperature over Ω_S

Ω_S = Source size, solid angle in steradians

$\Omega_A = \dfrac{4\pi}{G}$ Antenna beam size, solid angle in steradians

G = Antenna gain as a power ratio

The source solid angle may be found by:[3]

$$\Omega_S = 2\pi \left(1 - \cos\frac{\delta_S}{2} \right) \qquad\qquad \text{(Eqn 5A)}$$

where

Ω_S = Source solid angle in steradians

δ_S = Source angular diameter; e.g., arcminutes or radians

If we assume that the source is spherical, it is a simple matter of trigonometry via the cosecant function to derive an equation for Ω_S versus source linear size and distance.

$$\Omega_S = 2\pi \left[1 - \cos\left(\sin^{-1}\frac{D}{2r} \right) \right] = 2\pi \left(1 - \sqrt{1 - \frac{D^2}{4r^2}} \right) \qquad\qquad \text{(Eqn 5B)}$$

where

Ω_S = Source solid angle in steradians

D = Source linear diameter; e.g., km

r = Distance to source; e.g., km

Combining equations 1, 3, and 5B, we can rewrite the small source antenna temperature in terms of source temperature, linear size, and distance:

$$T_{ANT} = \frac{G}{2}\left[1 - \cos\left(\sin^{-1}\frac{D}{2r}\right)\right]T_{SRC} \qquad \text{(Eqn 6)}$$

where

$G =$ Antenna gain power ratio

$D =$ Source linear diameter; e.g., km

$r =$ Distance to source; e.g., km

$T_{SRC} =$ Source average temperature

Unavoidably, the antenna will also respond to the galactic background radiation. For the galactic background, it is useful here to approximate $T_S(\theta,\phi)$ with a constant minimum temperature T_{GB}; i.e., what an antenna having a beam size on order of Ω_A will see when aimed toward the galactic poles.

Similarly, the normalized antenna pattern $P_N(\theta,\phi)$ integrated over the whole sky is equal to Ω_A if the antenna has no side lobes. If side lobes are present, then the total Ω_A is shared between the lobes and the gain of the main beam is lower than it would be otherwise. Here we calculate Ω_A from the gain of the main beam, not by integrating the antenna pattern over the whole sky, so we arrive at Ω_A for the main beam, not for the whole antenna pattern.

For the galactic background (large source, $\Omega_S \geq \Omega_A$), Equation 2 simplifies to:

$$T_{ANT} = T_{GB} \qquad \text{(Eqn 7)}$$

where

$T_{GB} =$ Minimum average galactic background temperature

The antenna will see the sum of the contributions from the small source and the galactic background.

$$T_{ANT} = \frac{\Omega_S}{\Omega_A}T_{SRC} + T_{GB} \qquad \text{(Eqn 8)}$$

where

$\Omega_S = $ Source size, solid angle in steradians

$\Omega_A = $ Antenna beam size, solid angle in steradians

$T_{SRC} = $ Source average temperature over Ω_S

$T_{GB} = $ Minimum average galactic background temperature

The observed temperature will include a contribution from the receiver's circuitry; however, the Jove receiver's noise temperature is about a hundredth that of the galactic background at 20 MHz, so we may safely ignore the receiver's contribution.

To make a credible observation, the observed variation in antenna temperature must be several times greater than the RMS variation in the observed temperature. Note that the "signal" does *not* have to be hotter than the background, but merely larger than the *variation* in the background. That is, we want the "signal" to be several standard deviations above the mean.

The RMS noise variation (i.e., one standard deviation) σ_T is given by: [4]

$$\sigma_T = \frac{T_{ANT}}{\sqrt{\Delta f \; \tau}}$$

(Eqn 9)

where

$T_{ANT} = $ Observed antenna temperature

$\Delta f = $ Pre-detection bandwidth of the receiver

$\tau = $ Detector integration time constant

The question arises: how many standard deviations is enough? For a mere visual determination that something *might* exist in a strip chart, perhaps one or two standard deviations would do it—but that would not be considered a reliable observation. If the signal is three standard deviations from the mean, that's a minimally good observation. The NRAO suggests five standard deviations. [4] This awards the observer with ironclad evidence of an event. We shall thus use five standard deviations as our requirement.

Evaluation

At 20 MHz, the minimum galactic background antenna temperature can be approximated by 50,000 K for antennas with very large beam widths—e.g., one or two dipoles.[5] The Radio Jove telescope has a bandwidth of about 6 kHz. The integration time constant—also known as the sample period—is usually set to 100 milliseconds. Given those values, the RMS variation in the galactic background is:

$$\sigma_T = \frac{T_{GB}}{\sqrt{\Delta f \ \tau}} = \frac{50,000\,\text{K}}{\sqrt{6,000\,\text{Hz} \ 0.1\,s}} \approx 2,000\,\text{K}$$

To make a valid observation of an event, we would need to see a change in observed antenna temperature of five times that, or 10,000 K.

Note that this assumes there is no RF interference (RFI). If RFI is present, then a valid observation requires seeing a change in antenna temperature about five times larger than the RMS variation in the total temperature observed, including the RFI.

At 20 MHz, the solar corona's brightness temperature is about 190,000 K and its apparent diameter is roughly 90 arcminutes, or three times the size of the visible 1.4 million km diameter solar disk.[6,7] From Equation 6, we find the antenna temperature contributed by emission from the solar corona:

$$T_{ANT} = \frac{G}{2}\left[1 - \cos\left(\sin^{-1}\frac{D}{2r}\right)\right]T_{SRC}$$

$$= \frac{10^{0.8}}{2}\left[1 - \cos\left(\sin^{-1}\frac{4.2\times10^6\,\text{km}}{2\left(1.5\times10^8\,\text{km}\right)}\right)\right](190,000\ \text{K}) \approx 59\,\text{K}$$

The contribution from the solar corona's emission is about 59 K, less than a hundredth of the 10,000 K required for a five sigma observation against the minimum galactic background.

We can calculate the antenna gain required to create a five sigma observation. Solving Equation 6 for antenna gain, we have:

$$G = \frac{2T_{ANT}}{\left[1 - \cos\left(\sin^{-1}\dfrac{D}{2r}\right)\right]T_{SRC}}$$

$$= \frac{2\,(10{,}000\,\text{K})}{\left[1 - \cos\left(\sin^{-1}\dfrac{4.2\times10^6\,\text{km}}{2\left(1.5\times10^8\,\text{km}\right)}\right)\right](190{,}000\,\text{K})} \approx 1100 \approx 30\,\text{dBi}$$

This is a very high gain for an HF telescope. The largest HF radio telescopes are able to achieve this, but it is not possible with a few dipoles.

We can calculate what the solar corona's brightness temperature would need to be in order to make it observable with a Radio Jove telescope. Solving Equation 6 for T_{SRC} we have:

$$T_{SRC} = \frac{2T_{ANT}}{G\left[1 - \cos\left(\sin^{-1}\dfrac{D}{2r}\right)\right]}$$

$$= \frac{2\,(10{,}000\,\text{K})}{\left(10^{0.8}\right)\left[1 - \cos\left(\sin^{-1}\dfrac{4.2\times10^6\,\text{km}}{2\left(1.5\times10^8\,\text{km}\right)}\right)\right]} \approx 32\,\text{million K}$$

This is much hotter than the solar corona's known brightness temperature.

We can calculate how close to the Sun the telescope would need to be to see a five sigma deviation due to the solar corona passing through the beam. Solving Equation 6 for distance:

$$r = \frac{D}{2\sin\left[\cos^{-1}\left(\dfrac{2\,T_{ANT}}{G\,T_{SRC}} - 1\right)\right]}$$

$$= \frac{4.2\times10^6\,\text{km}}{2\sin\left[\cos^{-1}\left(\dfrac{2\,(10{,}000\,\text{K})}{10^{0.8}\,(190{,}000\,\text{K})} - 1\right)\right]} \approx 1.2\times10^7\,\text{km} \approx 0.08\,\text{AU}$$

This is about one fifth of Mercury's distance from the Sun.

What if we did not use a completely stock Radio Jove telescope? We might be able to open up the Jove receiver's bandwidth to perhaps 50 kHz and use an integration time constant of 10 seconds. The RMS variation in the galactic background would then be 71 K instead of 2,000 K, so we'd look for a five sigma observation of 350 K. To see 350 K from the solar corona, we'd need an antenna gain of about 16 dBi.

An array of sixteen dipoles has roughly 17 dBi gain, enough for the job. Building such an array is a large project, but feasible for an amateur radio astronomer. It would be an interesting experiment to see if one could separate the 350 K variation as the Sun crosses the beam from the normal variations in daytime band noise.

Conclusion

The quiet Sun is not observable at 20 MHz with a Radio Jove radio telescope; the emission is too weak by at least two orders of magnitude. However, with a larger antenna array and a few modifications to the Jove receiver, the solar corona may be observable.

References

[1] Kraus, J., Antennas, 2nd edn., (1988), sec. 2-22.

[2] Reyes, F., *Radio Telescope Response from Extended and Point Sources* (course notes) (2012).

[3] Wolfram Mathworld – Spherical Cap, eqn 16 (accessed 2013)
http://mathworld.wolfram.com/SphericalCap.html

[4] Condon, J. & Ransom, S. NRAO Essential Radio Astronomy – Radiometers (2010).
http://www.cv.nrao.edu/course/astr534/Radiometers.html

[5] Typinski, D., *The Galactic Background in the Upper HF Band*, (2013).

[6] Brazhenko, A., et al., *Peculiarity of Continuum Emission from the Upper Solar Corona at Decameter Wavelengths*, (2012).

[7] NASA/GSFC, Sun Fact Sheet (accessed 2013).
http://nssdc.gsfc.nasa.gov/planetary/factsheet/sunfact.html