~ A Different Kind of Solar Power ~

by Dave Typinski, AJ4CO

Our Sun, having spent the last year and a half in the doldrums, is finally becoming slightly active. This may herald the arrival of the much anticipated and long delayed solar cycle 24. In evidence thereof, I present a strip chart and spectrogram of a solar radio emission event I recorded on July 2nd, 2010. Using this event as an example, a discussion of what antenna temperature is and how it can be used to calculate emission source power is presented.

The Observatory

I use two RadioJove (RJ) direct conversion receivers tuned 300 kHz apart, one at 19.95 MHz and the other at 20.25 MHz.[1] The audio outputs of these receivers are sent to Radio-SkyPipe (RSP), a strip chart program that uses a PC's sound card as an analog to digital converter.[2] I also use an SDR-14 that feeds a spectrograph application, either Spectrograph or Spectravue.[3,4,5] The RJ receivers and the SDR-14 are fed simultaneously through power splitters by an RJ dual-dipole array phased to look directly south with an elevation angle of ~56°.[6] Figure 1 is a block diagram of my observatory equipment configuration.

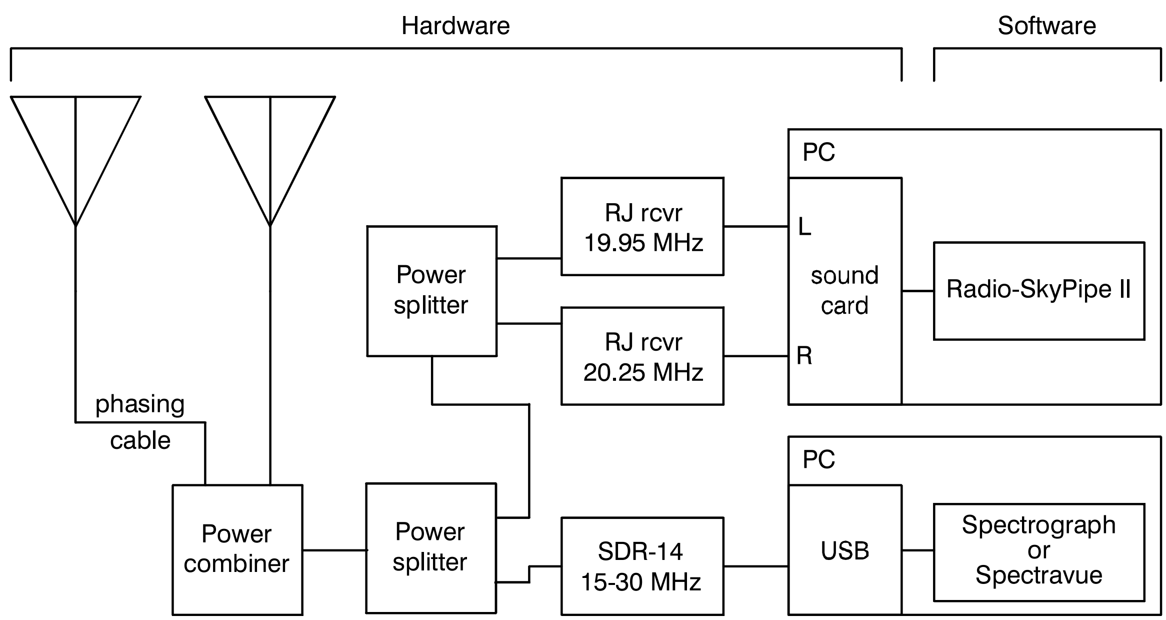

Figure 1 – AJ4CO Observatory station architecture.

In this arrangement, a portion of the local oscillator signals from the two RJ receivers leaks back through the power splitters into the SDR-14. This is advantageous. The RJ receivers do not have frequency displays and tend to drift in frequency from day to day by few kHz; however, I can use the SDR-14 and Spectravue to obtain a measurement of the receivers' frequencies at any given time.

The Observation

The Sun emits many kinds of bursts of energy from radio waves to X-rays. On July 2nd, 2010 at approximately 1717 UTC, I observed a radio emission from the Sun. Figures 2 and 3 show the data recorded by RSP and Spectrograph. This solar event was cataloged number 5000 by the NOAA Space Weather Prediction Center.[7, 8]

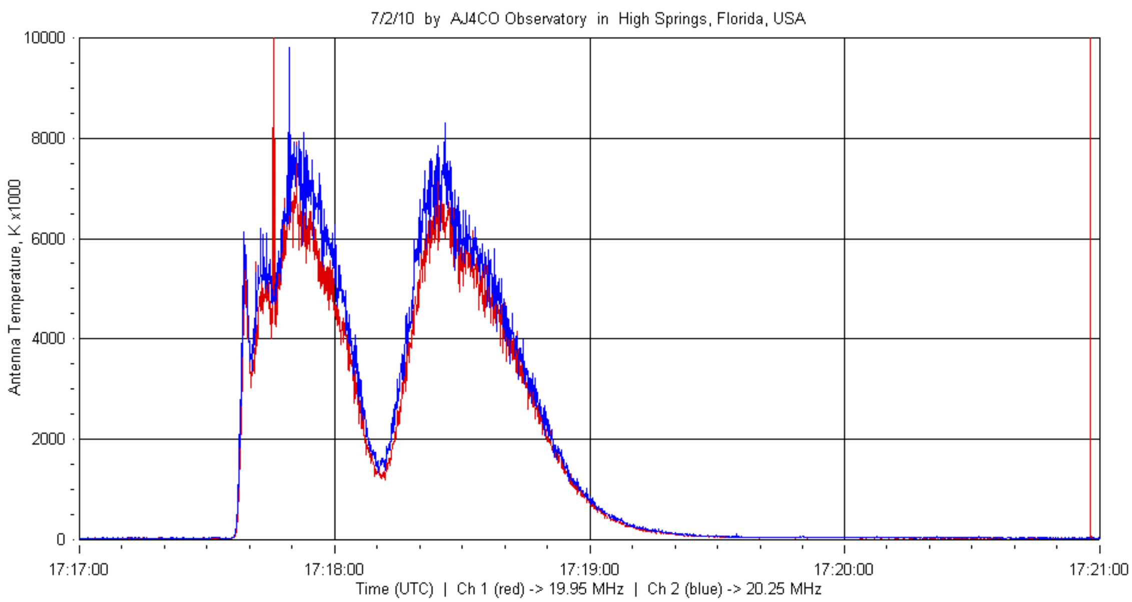

Figure 2 – Strip chart showing a solar radio burst.

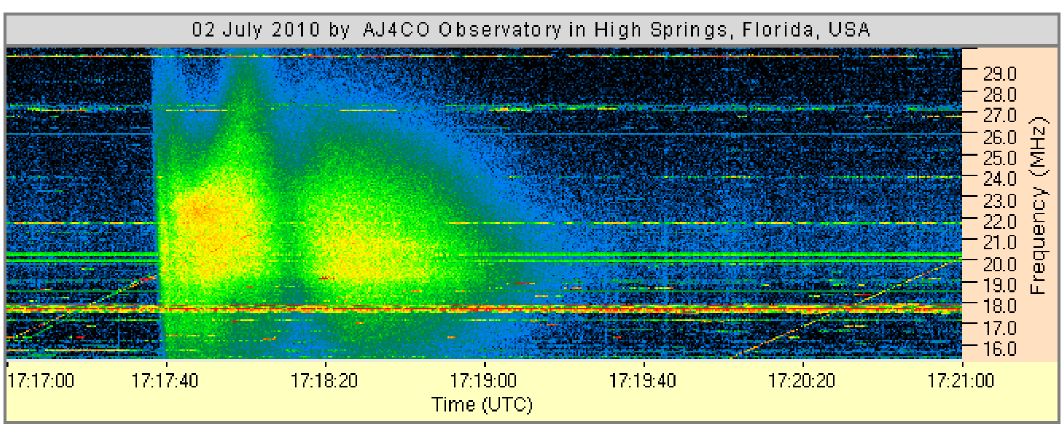

Figure 3 – Spectrogram showing a solar radio burst. Note that the structure in frequency is partially due to the dipole's electrical characteristics.

Antenna Temperature

A question that often arises when discussing this with people not versed in radio astronomy is, "How can your antenna be at 7 million K? Awfully warmish for copper, isn't it?" I know that was the primary question I had when I first saw one of these strip charts.

In radio astronomy, there are two common ways of communicating signal strength.

One way is by stating signal strength in terms of flux density (also known as specific flux), or watts per square meter per hertz, with the understanding that there exists a given center frequency and a given bandwidth. The common unit is the jansky (Jy), which is defined as 10^{-26} $W \cdot m^{-2} \cdot Hz^{-1}$. Signals from objects outside the solar system can be as strong as a few thousand jansky (e.g., the radio galaxy Cygnus A), but most are far weaker than one jansky. On the other hand, signals from the Sun and Jupiter can be stronger than a million jansky.

The other way to indicate signal strength is to state it in terms of antenna temperature in kelvin. This is the temperature that an imaginary resistor at the antenna terminals would have to possess in order to create a thermal noise signal of the same magnitude as that observed by the receiver. Antenna temperature, like flux density, assumes there exists a center frequency and bandwidth over which the signal is measured.

What can we say about a solar radio burst producing an apparent 7 MK temperature in my antenna?

Noise Power

First, we can say that signals received from the Sun are simply noise. As such, the antenna produces electrical power at the antenna terminals as if it were a resistor at some temperature *T*. We can convert that antenna temperature to power via the Rayleigh-Jeans approximation to the Planck law [9], which states

$$P = kTB \tag{1}$$

where

k = Boltzmann's constant = 1.381×10^{-23} $W \cdot s \cdot K^{-1}$

T = temperature (in our case, antenna temperature) in kelvin (K) = 7×10^{6} K

B = receiver bandwidth in hertz (Hz)

In my case, the RJ receivers have a bandwidth of 6 kHz [10], so

$$\boxed{P = \left(1.381\times10^{-23}\ \text{W}\cdot\text{s}\cdot\text{K}^{-1}\right)\left(7\times10^{6}\ \text{K}\right)\left(6\times10^{3}\,\text{Hz}\right) = 5.8\times10^{-13}\ \text{W} = 580\ \text{femtowatts}}$$

What does 580 femtowatts mean? That's the power at the antenna's feed point being fed, via the feed line, into the 50 ohm load of the receiver's input amplifier. We can use this to figure out the power of the emission source, but we have to go through a few steps first.

Flux Density

If we know something about the antenna, we can say something about the signal's flux density—its power per unit area per unit bandwidth—also known as specific flux or S_ν. Flux density is often cited in units of jansky (Jy), defined as 10^{-26} W·m^{-2}·Hz^{-1}. Flux density is related to power via [11]

$$S_\nu = \frac{P}{A_{eff}\, B} \tag{2}$$

where

P = power (W)

A_{eff} = antenna effective area (m^2)

B = receiver bandwidth in hertz (Hz)

But wait, "antenna effective area"? What's that?

Antenna Area

The effective area of an antenna is [12]

$$A_{eff} = \frac{G\,\lambda^2}{4\pi} = \frac{G\,c^2}{4\pi\, f^2} \tag{3}$$

where

G = antenna gain

λ = observation wavelength (m)

c = speed of light = 2.998×10^8 m·s^{-1}

f = observation frequency (Hz)

Great, so what's antenna gain?

Antenna Gain

The gain of an antenna, denoted by G, is the ratio of the antenna's ability to emit and absorb RF radiation to the ability of an isotropic radiator to do the same. A convenient way to state G is in terms of dBi where

$$G = \left(\frac{P}{P_i}\right) \quad \text{and} \quad dBi = 10\log\left(\frac{P}{P_i}\right) = 10\log G \tag{4}$$

where

P_i = The power available at the terminals of an isotropic radiator when receiving a given signal

P = The power available at the terminals of the antenna when receiving the same given signal

An isotropic radiator, by definition, has a gain of 0.0 dBi and covers the whole sky. A positive gain is obtained at the expense of angular coverage. A Yagi antenna, for example, might have a 14 dBi gain at its design frequency in the direction of maximum gain with a half power beam width of 25°. The 70-meter dishes of the Deep Space Network have gains of about 80 dBi in the Ka band (31 to 38 GHz) and a half power beam width of around three arcseconds. [13]

In my case, I'm using a simple two-dipole phased array designed for 20.1 MHz. I have modeled this array in EZNEC+. [14] Theoretically, the array has a maximum gain of 7.6 dBi and a half power beam width of 111° in azimuth and 64° in altitude. The beam center is fixed at 180° in azimuth and 56° in altitude.

The Sun at the time of observation was at 161° azimuth and 83° altitude, putting it at roughly the -2.5 dBi of peak area of the beam. As such, for this observation, the antenna has a gain of 7.6 dBi – 2.5 dBi ≅ 5 dBi.

Before we can use that, however, we have to convert from decibels to the power ratio G.

Since $dBi = 10\log G$ from Eqn. 4 above, the gain G is simply $10^{\left(\frac{dB}{10}\right)}$, or in my case,

$$\boxed{G = 10^{\left(\frac{5}{10}\right)} \cong 3.2}$$

Antenna Area Redux

We can now find the array's effective area. From Eqn. 3 above,

$$A_{eff} = \frac{G\lambda^2}{4\pi} = \frac{G c^2}{4\pi f^2}$$

where

G = antenna gain = 3.2
c = speed of light = 2.998×10^8 m·s^{-1}
f = observation frequency = 20.1 MHz

$$\boxed{A_{eff} = \frac{(3.2)\left(2.998\times10^8\,\mathrm{m{\cdot}s^{-1}}\right)^2}{4(3.14)\left(2.01\times10^7\,\mathrm{Hz}\right)^2} = 56.7\,\mathrm{m}^2}$$

Note that I used the average of the two receivers' frequencies. The 150 kHz difference between each receiver's frequency and the average produces a ~2% difference in antenna area; this is small enough to ignore since my strip chart temperature reading is only good to ~5%.

Flux Density Redux

We can now solve for flux density. From Eqn. 2 above,

$$S_\nu = \frac{P}{A_{eff}\,B}$$

where
P = power = 5.8×10^{-13} W
A_{eff} = antenna effective area = 56.7 m^2
B = receiver bandwidth = 6 kHz

$$\boxed{S_\nu = \frac{P}{A_{eff}\,B} = \frac{5.8\times10^{-13}\,\mathrm{W}}{\left(56.7\ \mathrm{m}^2\right)\left(6\times10^3\,\mathrm{Hz}\right)} = 1.7\times10^{-18}\,\mathrm{W{\cdot}m^{-2}{\cdot}Hz^{-1}} = 170\ \text{million Jy}}$$

So, there's our answer: a 7 MK antenna temperature *in my antenna* indicates a source flux density of 170 million jansky. It's worthy to note that this may not be the case in someone else's antenna because their antenna area and antenna gain may be different.

It is apparent from the fact that we used bandwidth twice, once for noise power and once for flux density, that we should be able to ignore bandwidth if we go straight from antenna temperature to flux density—but that would have robbed us of the opportunity to discuss power.

We have $S_\nu = \dfrac{P}{A_{eff}\,B}$ and $P = kTB$

which means

$$S_\nu = \frac{k\,T}{A_{eff}} \qquad (5)$$

where
k = Boltzmann's constant = 1.381×10^{-23} W·s·K^{-1}
T = temperature = 7×10^6 K
A_{eff} = antenna effective area = 56.7 m^2

$$\boxed{S_\nu = \frac{k\,T}{A_{eff}} = \frac{\left(1.381\times10^{-23}\,\mathrm{W{\cdot}s{\cdot}K^{-1}}\right)\left(7\times10^6\,\mathrm{K}\right)}{56.7\,\mathrm{m}^2} = 1.7\times10^{-18}\,\mathrm{W{\cdot}m^{-2}{\cdot}Hz^{-1}} = 170\ \text{million Jy}}$$

Knowing the signal's flux density at my antenna, what can we do with it?

Emission Source Power

If we *assume* the source radiated this energy isotropically—i.e., in all directions evenly—then we can find out how much power it was producing. This may *not* be a safe assumption for astronomical sources; the RF energy is often emitted in a beam by electrons interacting with magnetic fields—e.g., cyclotron and synchrotron radiation. Nevertheless, let's pretend it was isotropic and see what happens.

The Earth—and hence my antenna—is 1 AU from the Sun. A sphere of radius 1 AU has an area of $4\pi AU^2$, or about 2.8×10^{23} m^2.

My antenna saw a flux density of 1.7×10^{-18} W·m^{-2}·Hz^{-1}. If we assume the source on the Sun evenly illuminated this whole 1 AU radius sphere, then the power per unit frequency is:

$$(1.7\times10^{-18}\ \mathrm{W \cdot m^{-2} \cdot Hz^{-1}})(2.8\times10^{23}\ \mathrm{m^2}) = 480\ \mathrm{kW \cdot Hz^{-1}}.$$

My chart doesn't show it, but according to the NOAA Space Weather Prediction Center, this particular emission event was recorded by their observatories at 245 MHz at 1.9 million Jy.[15] There is no data for the bandwidth of this particular burst. We can, however, see from my spectrogram that it definitely spanned down to 15 MHz, and from the calculations above, it was quite a bit stronger at these longer wavelengths.

Solar bursts often start at high frequencies and move low, so the instantaneous emission bandwidth may not have spanned at the 230 MHz from 15 MHz to 245 MHz. It's also worthy to note that the flux density wasn't flat across that range. The flux density was roughly a hundred times greater at 20 MHz than it was at 245 MHz.

We can, however, use the spectrogram in Figure 3 to obtain an estimate of the instantaneous bandwidth. It appears that the average power extends from 17 to 25 MHz. Although this is undoubtedly due to the electrical characteristics of the dipole antennas and not the burst itself, we shall use this 8 MHz bandwidth and see what happens.

To find the total power of the emission source, we simply multiply the flux density times the bandwidth:

$$(480\ \mathrm{kW \cdot Hz^{-1}})(8\ \mathrm{MHz}) = 3.8\ \text{Terawatts}.$$

However, we need to make some corrections to that figure.

A dipole antenna a quarter wavelength off the ground, like mine, has a radiation efficiency of only 90%.[16] That changes the emitted power to

$$\frac{3.8\,\mathrm{TW}}{0.9} = 4.2\,\mathrm{TW}$$

Then, add the fact that this was a mid-day observation, meaning the ionosphere's F layer was relatively thick, which means it will absorb and reflect some of the incoming solar 20 MHz radiation back out into space in much the same way it reflects and absorbs terrestrial signals back to Earth. During the day, the ionosphere will attenuate an extraterrestrial 20 MHz signal by roughly 3 dB (a factor of two).[17]

To find the source power, we have

$$\boxed{\text{Emission source total estimated power} = 2\left(4.2\,\text{TW}\right) = 8.4\,\text{TW}}$$

By comparison, the world's total installed electricity generating capacity is only about 4 TW.[18]

Looking at the strip chart, we see two peaks, each lasting approximately 5 seconds. We can thus approximate the total energy released during the two instances of peak emission.

$$\boxed{E_{peak} = 8.4\,\text{TW}\left(10\,\text{s}\right) = 8.4\times10^{13}\,\text{J} = 84\ \text{TJ}}$$

Given that 1 kilotonne TNT nuclear equivalent is 4.184×10^{12} J, we can find the energy released in terms of nuclear weapons.

$$\boxed{E_{peak} = 8.4\times10^{13}\,\text{J}\left(\frac{1\,\text{kT}}{4.184\times10^{12}\,\text{J}}\right) = 20\,\text{kT}}$$

So, at its peak, we find—*very* approximately!—that this solar burst released as much energy in 10 seconds as did the detonation of the Fat Man nuclear weapon over Nagasaki.

It must be said again: these numbers are only very coarse approximations, full of assumptions about the emission source that may or may not be true. To get a more accurate picture of the total power and energy involved, one would need to have an idea of the emission source's beam width, how the flux density evolved in time across all frequencies. We could then integrate flux density with respect to frequency and time to obtain power for an isotropic source, then de-rate that given a less than isotropic beam profile—if in fact the beam profile didn't also change in time. If it did, then that too must be taken into account to find the true source power and energy.

The problem is, none of these solar emission characteristics are well known. Continuous spectra across all frequencies does not exist for solar emission events (the STEREO/WAVES spectrographs cover 10 kHz to 16 MHz).[19] We have even less of an idea about the emission beam pattern: there are few radio observatories along the ecliptic measuring these emissions and none appreciably above or below the ecliptic. Thus, while this article is a fun exercise for the sake of showing what can be done and how to do it, the total energy and total power numbers are not very reliable.

This was a medium size burst as these things go. The strongest solar emissions produce antenna temperatures over thirty times hotter (~250 MK), last several times longer, and cover a much wider bandwidth.

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