SECTION 3

THEORY OF OPERATION

Note: For the reader who is familiar with the theory of time compression analyzers or the reader who is not interested in the theoretical details, this section can be skipped in its entirety.

3.1 GENERAL

3.1.1 This section provides the background theory of operation for SAICOR's time compression spectrum analysis equipment. This theory may be useful to the operator in understanding the processing of signals by the SAI-51A or SAI-53A in its various modes.

The spectral or frequency domain properties of signals contain valuable information which can be used as an important design or diagnostic and interpretative tool. Various techniques for obtaining the spectrum of signals will be described. The more conventional procedures are compared with the time compression technique. Then there is a description of the three most important parameters (sampling, signal length, and memory size) which enter into the determination of the spectrum using time compression techniques. This is followed by a description of digital averaging and then a block diagram discussion of the SAI-51A and SAI-53A.

3.2.1 SPECTRAL ESTIMATES. The mathematical model which is applicable for making estimates of the power contained in a signal within a bandwidth of B_R Hz is shown in figure 3.1.

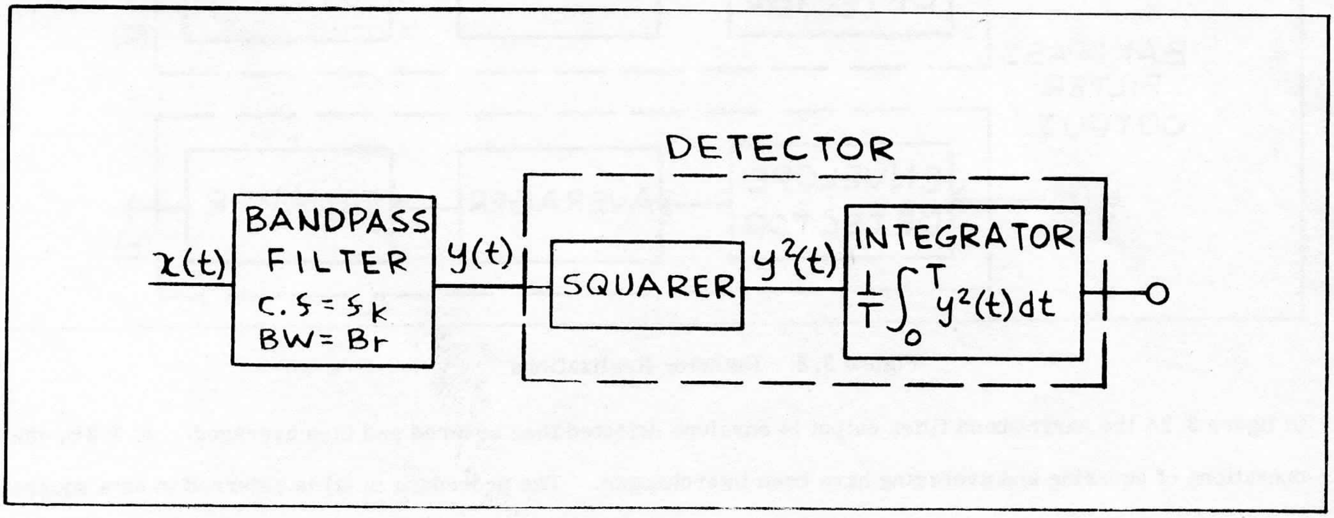

Figure 3.1: Power Measurements

The signal x(t) is passed through a narrowband bandpass filter centered at f_k Hz and having a bandwidth of B_R Hz. This filter output is then applied to a detector which consists of a squarer and an averager. The detector output yields an estimate of the signal power in this bandwidth. This estimate improves as the averaging time increases. Conventionally this averaging time is related to multiples of the reciprocal of the filter bandwidth B_R. Thus the parameter $B_R T$ is used as a performance criterion for these measurements. The interrelationships between these spectral estimates and the concept of degrees of freedom will be discussed in detail in section 3.4.1. Qualitatively, the interplay between the parameters B_R and T can be explained as follows: The signal output of the bandpass filter is constrained to a bandwidth of B_R Hz. From the sampling theorem for bandpass signals all the information contained in the waveform can be determined from magnitude and phase samples (or co and quadrature components) taken at the rate of B_R samples per second (each). Samples taken at a higher rate do not add any additional information. Therefore, $2B_RT$ numbers per sec (B_R phase and B_R magnitude) represent in effect all the information contained. Because $2B_RT$ numbers per sec (B_R phase and B_R magnitude) can be used to fully represent the signal information, the averager, although depicted as an analog device, can be viewed as a digital device which is adding independent numbers. Since the output of the squarer is proportional to the magnitude squared of the signal, the averager can be considered to be adding independent numbers at the rate of B_R per second (that is, the phase information is lost). For an averaging time of $T=1/B_R$ no averaging takes place (B_RT=one number). For $T=K/B_R$, K numbers are effectively added in the averager. (section 3.4.1 relates this addition process to the concept of degrees of freedom). Other realizations of the detector of figure 3.1 are shown in figure 3.2.

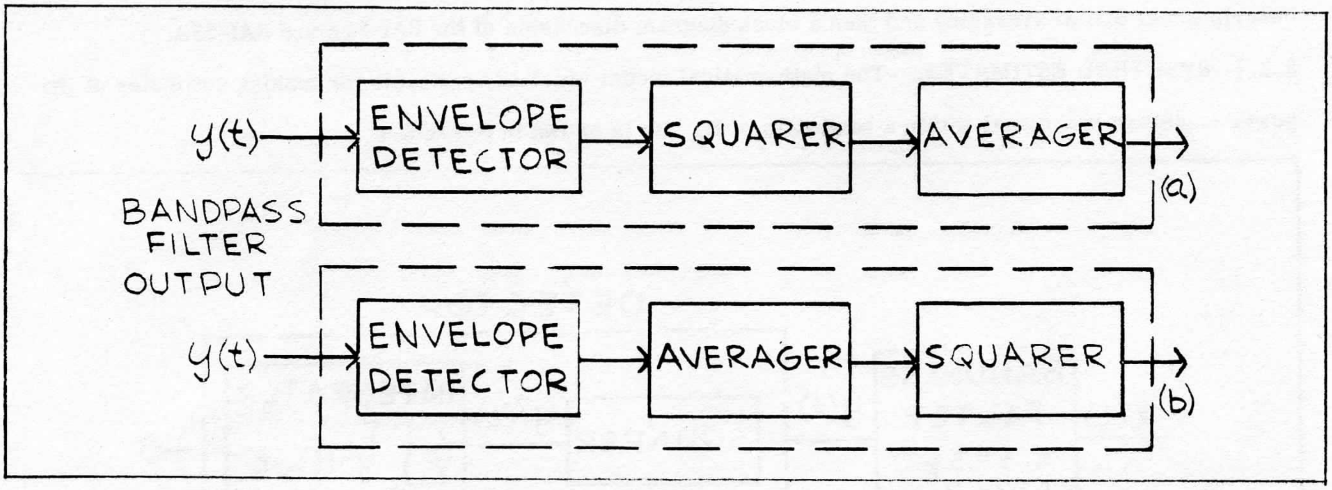

Figure 3.2: Detector Realizations

In figure 3.2A the narrowband filter output is envelope detected then squared and then averaged. In 3.2B, the operations of squaring and averaging have been interchanged. The procedure in (a) is referred to as a square-law envelope detector and is essentially identical to the model of 3.1. The procedure in (b) allows the averager to process a "linear" signal (as opposed to a "squared" signal in(a)) and, therefore, imposes less stringent dynamic range problems on the overall system. In both systems, the detector output is proportional to the signal power (in the band B_R). However, the constant of proportionality is different for the two systems and must be properly considered when calibrated outputs are being measured.

Independent of the realization of the detector utilized and the use of a realizable bandpass filter in place of the ideal, the question to be posed is - CAN THIS CONCEPT BE EXTENDED TO MEASUREMENTS OF THE SIGNAL POWER OVER A GIVEN FREQUENCY RANGE ? THE PARALLEL FILTER APPROACH, WHICH WILL BE DESCRIBED NEXT, ANSWERS THIS QUESTION AFFIRMATIVELY.

3.2.2 A PARALLEL FILTER APPROACH

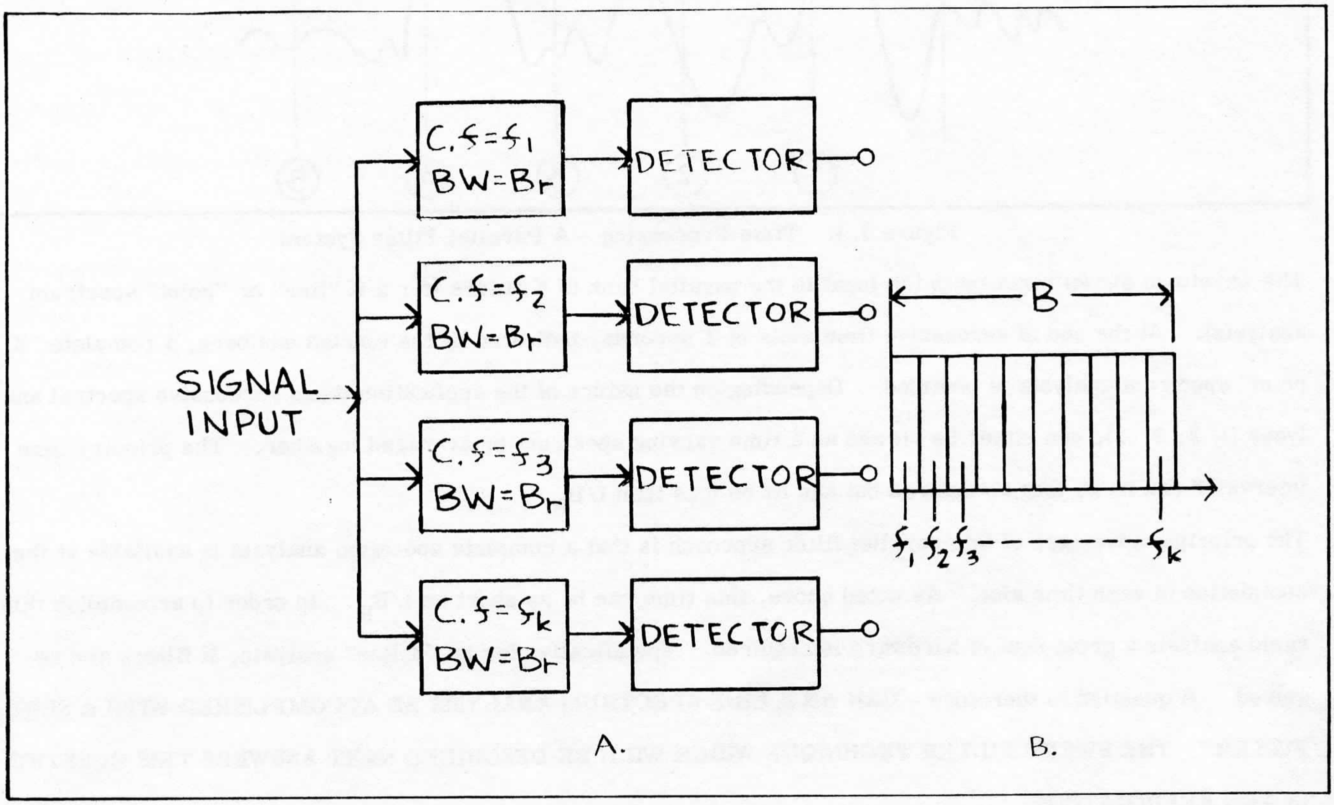

Figure 3.3: A Parallel Filter Technique

In the system of figure 3.3A, the input signal is applied in parallel to a set of K bandpass filters. Each filter has a bandwidth of B_R Hz and center frequencies f_K as shown in figure 3.3A. Therefore, a bandwidth of B Hz (0 to B Hz as shown in 3.3B) is covered by K filters where $K=B/B_R$. This system provides K "points" for the estimate of the power spectral density curve. Each point represents a measurement over a bandwidth B_R Hz. To improve the resolution requires more points on the curve and, therefore, a smaller B_R and more filters. As described previously, the minimum required integration time or processing time is related to $1/B_R$. Therefore, as the resolution improves, the signal length which must be processed increases. Figure 3.4 shows the capabilities of this parallel filter approach from a time process-point of view.

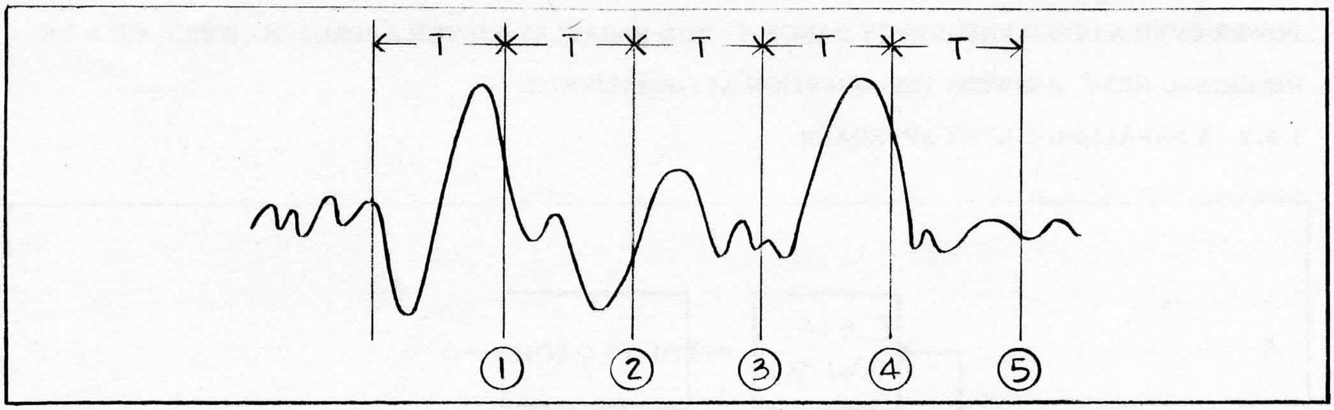

Figure 3.4: Time Processing - A Parallel Filter System

The waveform shown represents the input to the parallel bank of K filters (for a K "line" or "point" spectrum analysis). At the end of successive time slots of T seconds, indicated by the circled numbers, a complete "K point" spectrum analysis is obtained. Depending on the nature of the application these successive spectral analyses (1, 2, 3...), can either be viewed as a time varying spectrum or averaged together. The primary time interval T can be as long as desired but can be no less than $1/B_R$.

The principle advantage of this parallel filter approach is that a complete spectrum analysis is available at the completion of each time slot. As noted above, this time can be as short as $1/B_R$. In order to accomplish this rapid analysis a great deal of hardware is required. Specifically, for an "K line" analysis, K filters are required. A question is therefore - CAN AN K LINE SPECTRUM ANALYSIS BE ACCOMPLISHED WITH A SINGLE FILTER? THE SWEPT FILTER TECHNIQUE WHICH WILL BE DESCRIBED NEXT ANSWERS THIS QUESTION IN THE AFFIRMATIVE.

3.2.3 A SWEPT FILTER APPROACH

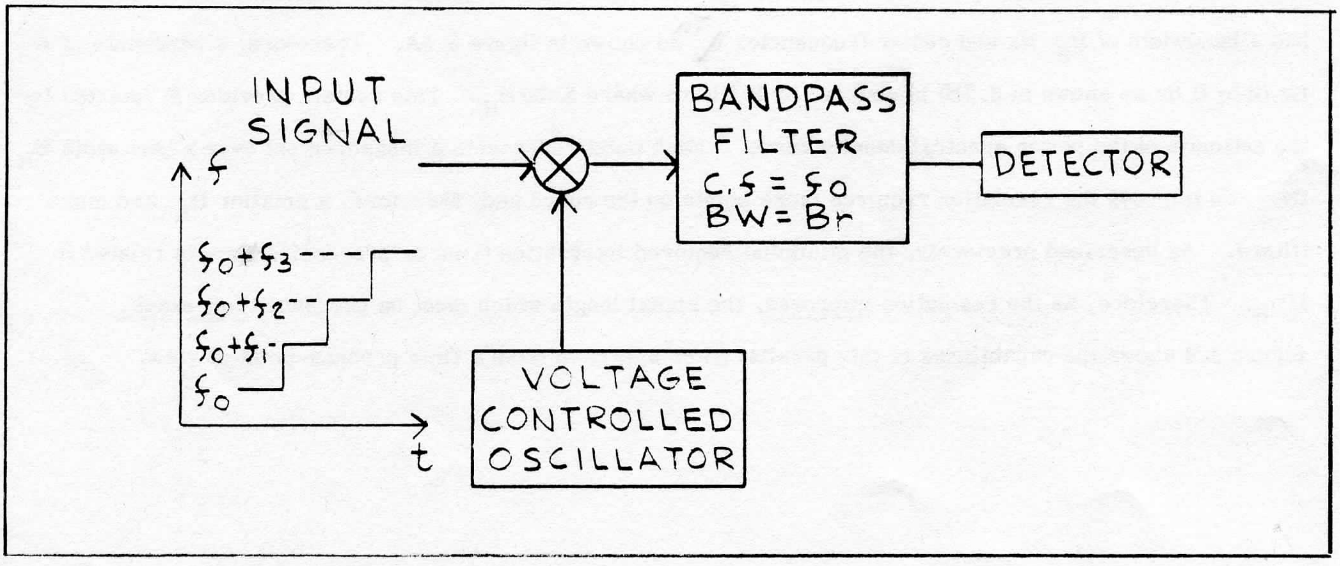

Figure 3.5: A Swept Filter Approach

In the system of figure 3. 5, the signal is applied to a mixer (multiplier) which is followed by a bandpass filter and detector identical to that described in figure 3. 1 and identical to one of the channels shown in the parallel processor of figure 3. 3. The second signal being applied to the mixer is a sine wave whose frequency vs. time curve is shown. The purpose of this stepped sine wave produced by the voltage controlled oscillator (VCO) is to sequentially shift or "sweep" the spectrum of the signal past a fixed analysis filter. Although it is the signal spectrum which is moved this technique is referred to as a swept filter or heterodyning approach.

It should be noted that the sinusoid remains fixed at any one frequency for a time T and as described previously; this time can be no shorter than $1/B_R$. In addition, although depicted as a simple heterodyning technique, practical constraints often cause this operation to be carried out in two stages of heterodyning to ease in the rejection of unwanted frequency components.

This swept filter approach has greatly reduced the required hardware but a sacrifice has been made in the processing time required to complete the spectrum analysis. This can be seen from figure 3. 6.

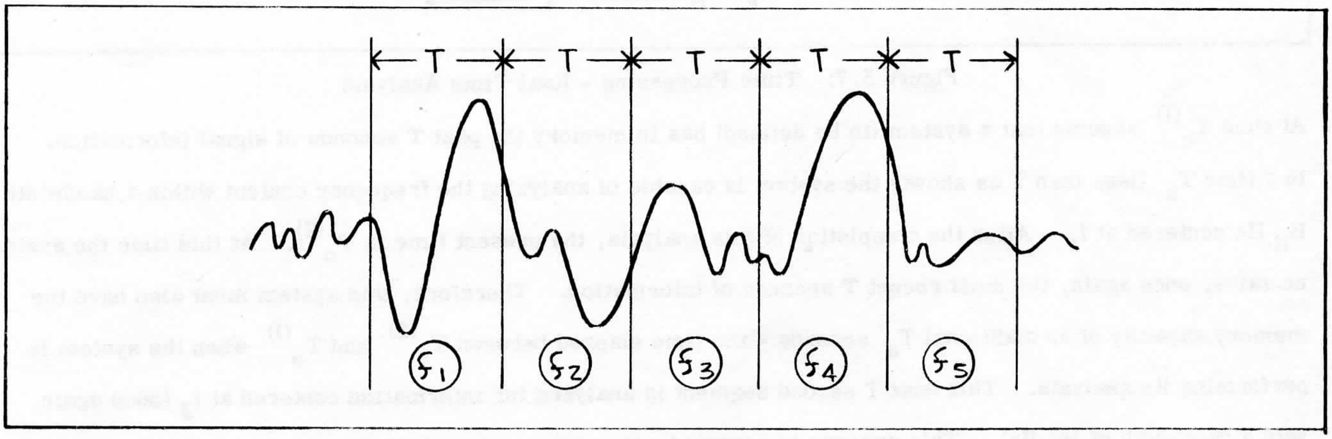

Figure 3. 6: Time Processing - A Swept Filter Approach

Since each frequency channel $f_1, f_2 \ldots f_K$ is examined sequentially for a time equal to T seconds, it takes KT seconds to complete a "K line" spectrum analysis. The minimum time is, therefore, K/B_R seconds for the complete analysis. Sometimes, the maximum sweep rate of this technique is referenced by noting that the minimum time per channel is $1/B_R$ and the channel separation is B_R. Therefore, the maximum sweep rate is B_R divided by $1/B_R^2$ (Hz/sec). As an example of the time required to perform a complete spectrum analysis consider a 200 line analysis and a resolution of 1 Hz (0-200 Hz total bandwidth). From the foregoing, the swept filter technique would require 200 seconds to complete one spectrum analysis since $T=1/B_R=1$ sec and K=200. A question is, therefore - CAN A SPECTRUM ANALYSIS BE PERFORMED IN A TIME COMPARABLE TO THAT OF THE PARALLEL FILTER APPROACH WITHOUT REQUIRING BUILDING A "BANK" OF FILTERS (EQUAL TO THE NUMBER OF LINES REQUIRED). THE TIME COMPRESSION TECHNIQUE TO BE DESCRIBED NEXT ANSWERS THIS QUESTION IN THE AFFIRMATIVE.

3.2.4 A TIME COMPRESSION APPROACH. Consider the time processing arrangement shown in figure 3.7.

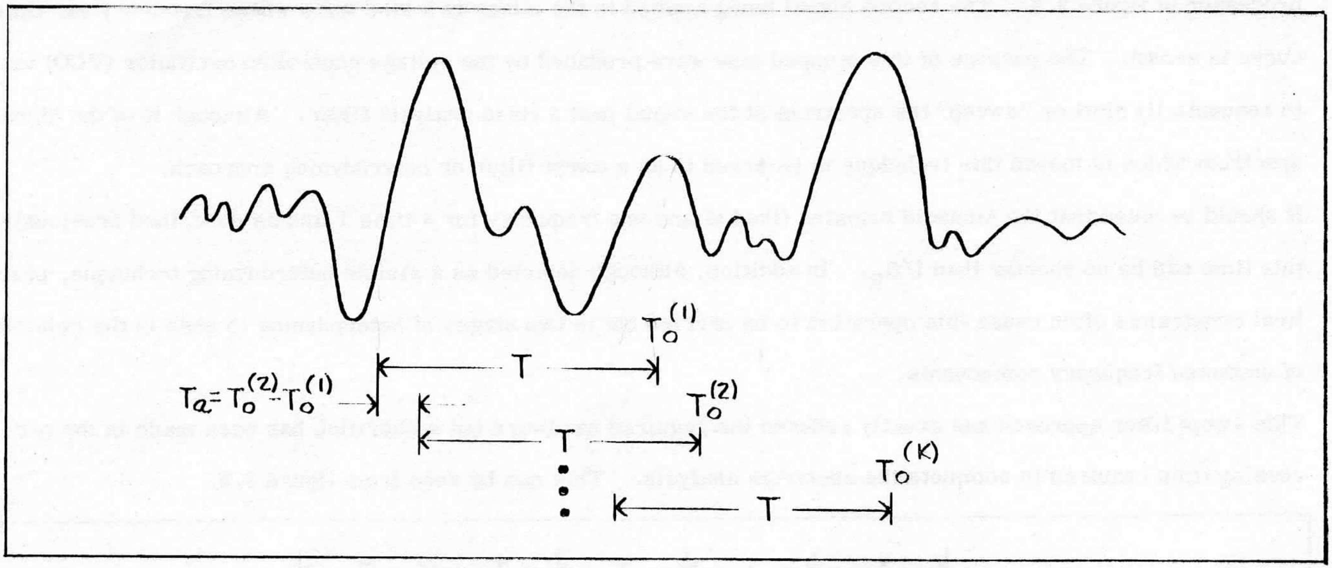

Figure 3.7: Time Processing - Real Time Analysis

At time $T_o^{(1)}$ assume that a system (to be defined) has in memory the past T seconds of signal information. In a time T_a (less than T as shown) the system is capable of analyzing the frequency content within a bandwidth B_R Hz centered at f_1. After the completion of this analysis, the present time is $T_o^{(2)}$. At this time the system contains, once again, the most recent T seconds of information. Therefore, this system must also have the memory capacity of an additional T_a seconds - the time elapsed between $T_o^{(2)}$ and $T_o^{(1)}$ when the system is performing its analysis. This next T second segment is analyzed for information centered at f_2 (once again with a bandwidth of B_R Hz). This process is repeated - the memory constantly being updated - until all the frequency channels have been analyzed. For a "K line" analysis, the total analysis time is KT_a [5]. (If desired, the K line analysis can then be repeated).

If KT_a=T, AS SHOWN IN FIGURE 3.7, THEN THE TIME TO PERFORM A COMPLETE (K LINE) SPECTRUM ANALYSIS IS EQUAL TO THE DATA COLLECTION TIME. THAT IS THE DEFINITION OF REAL TIME ANALYSIS. THAT IS, THE ANALYSIS TIME (FOR A COMPLETE SPECTRUM ANALYSIS) IS EQUAL TO OR LESS THAN THE DATA COLLECTION TIME.

In order to see the implication of this constraint consider figure 3.8.

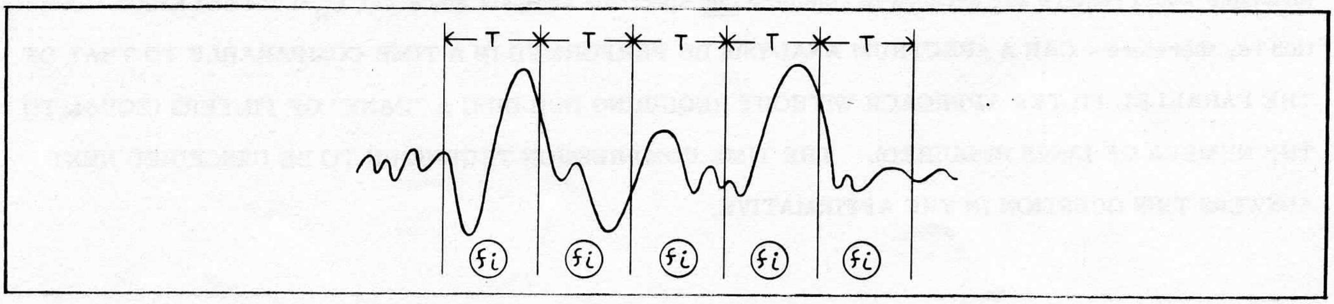

Figure 3.8: Real Time Processing

This diagram is obtained from figure 3.7 for $KT_a = T$. When this constraint is satisfied, the system is "ready" to analyze the band centered at any arbitrary center frequency f_i with data that is contiguous. Thus the analyzer is, in a sense breaking up the theoretical expression

$$\int_{-\infty}^{\infty} f(t)\exp\left\{-j2\pi f_i t\right\}dt = \int_0^T [\ ]\,dt + \int_T^{2T} [\ ]\,dt + \ldots$$

into T second segments.

When $KT_a < T$ the relationship is as shown in figure 3.9.

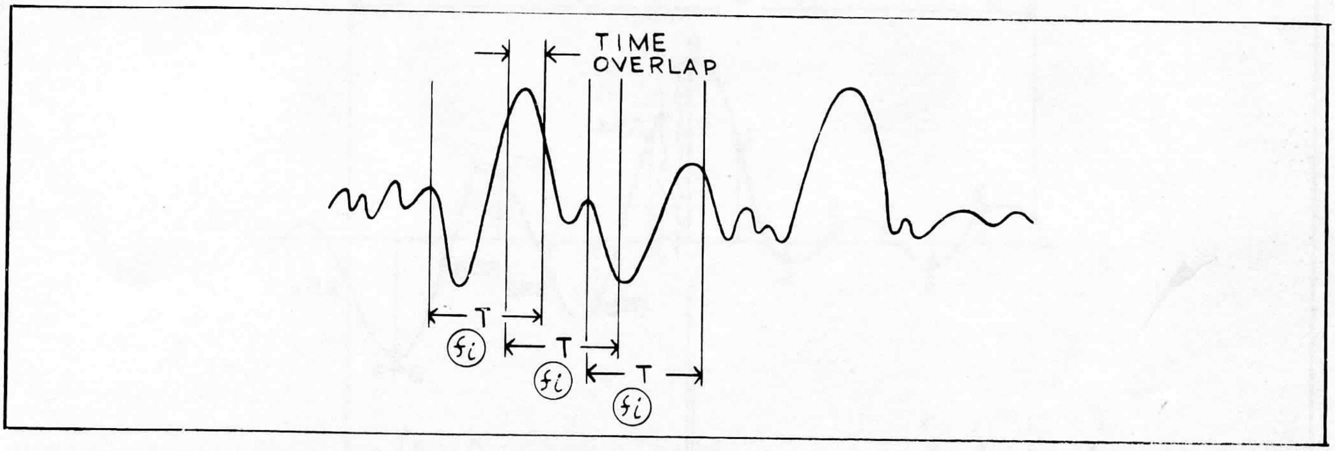

Figure 3.9: Processing with Time Overlap

This overlap in analysis segments is referred to as redundant processing and will be discussed in detail in section 3.4 where spectrum averaging is described.

When $KT_a > T$ "gaps" occur as shown in figure 3.10.

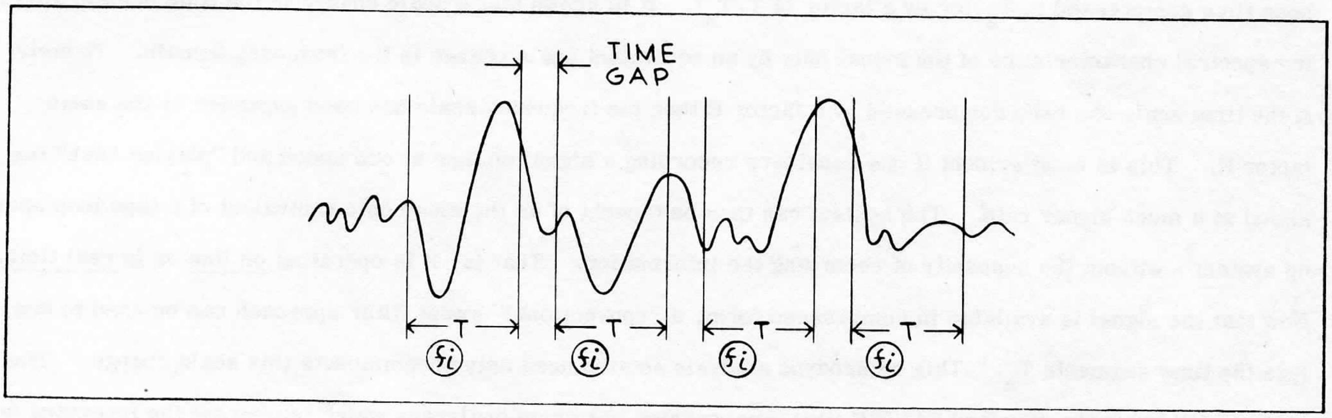

Figure 3.10: Processing with Time Gaps

The spectrum analysis can still be accomplished but it is possible to "lose" short bursts of information.

Up to this point, no mention has been made of the particular system required to perform the foregoing analysis. Several constraints, however, have been placed on the proposed system.

1. It must contain sufficient memory for T seconds worth of data (plus T_a seconds).

2. It must complete the analysis in a time T_a such that $KT_a < T$.

3. It performs the frequency analysis sequentially (ie. $f_1, f_2 \ldots f_K$).

From figure 3.7 one notes that at time $T_o^{(1)}$, T seconds of data are contained in the analyzer memory. However, the system must analyze frequency channel f_1 in T_a seconds where T_a is considerably less than T. Suppose this T seconds of data is available compressed in time as shown in figure 3.11.

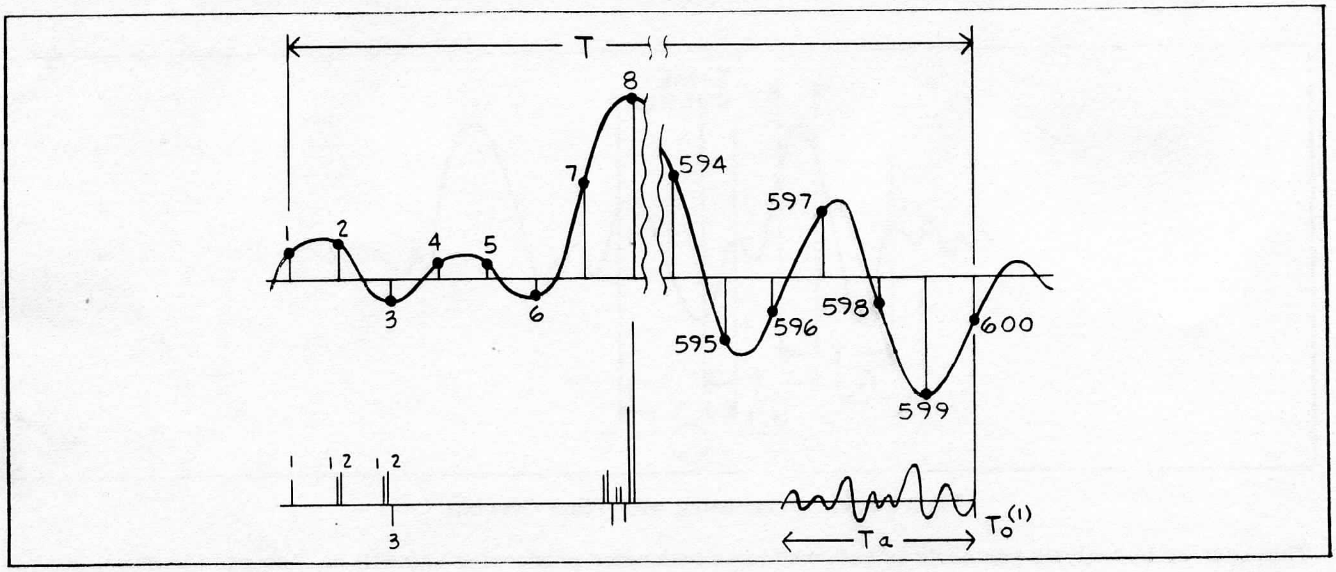

Figure 3.11: The Time Compression Approach

The signal that appears in 3.11 at time $T_o^{(1)}$ is identical (theoretically) to the original signal except that it has been time compressed to T_a (or by a factor of T/T_a). It is known that a scale change in the time domain alters the spectral characteristics of the signal only by an equivalent scale change in the frequency domain. Namely, if the time scale has been compressed by a factor R then the frequency scale has been expanded by the same factor R. This is most evident if one considers recording a signal on tape at one speed and "playing back" the signal at a much higher rate. The system can then be thought of as the electronic equivalent of a tape loop speed-up system - without the necessity of recording the information. That is, it is operating <u>on line</u> or in <u>real time.</u> Now that the signal is available in compressed form, a "conventional" swept filter approach can be used to analyze the time segments T_a. This heterodyne analysis section need only accommodate this scale change. The system utilized in the standard SAICOR time compression spectrum analyzers which implement the foregoing is shown in figure 3. 12.

As shown, the system can be divided into three sections;

1. The time compressor

2. The heterodyne analyzer

3. The digital averager

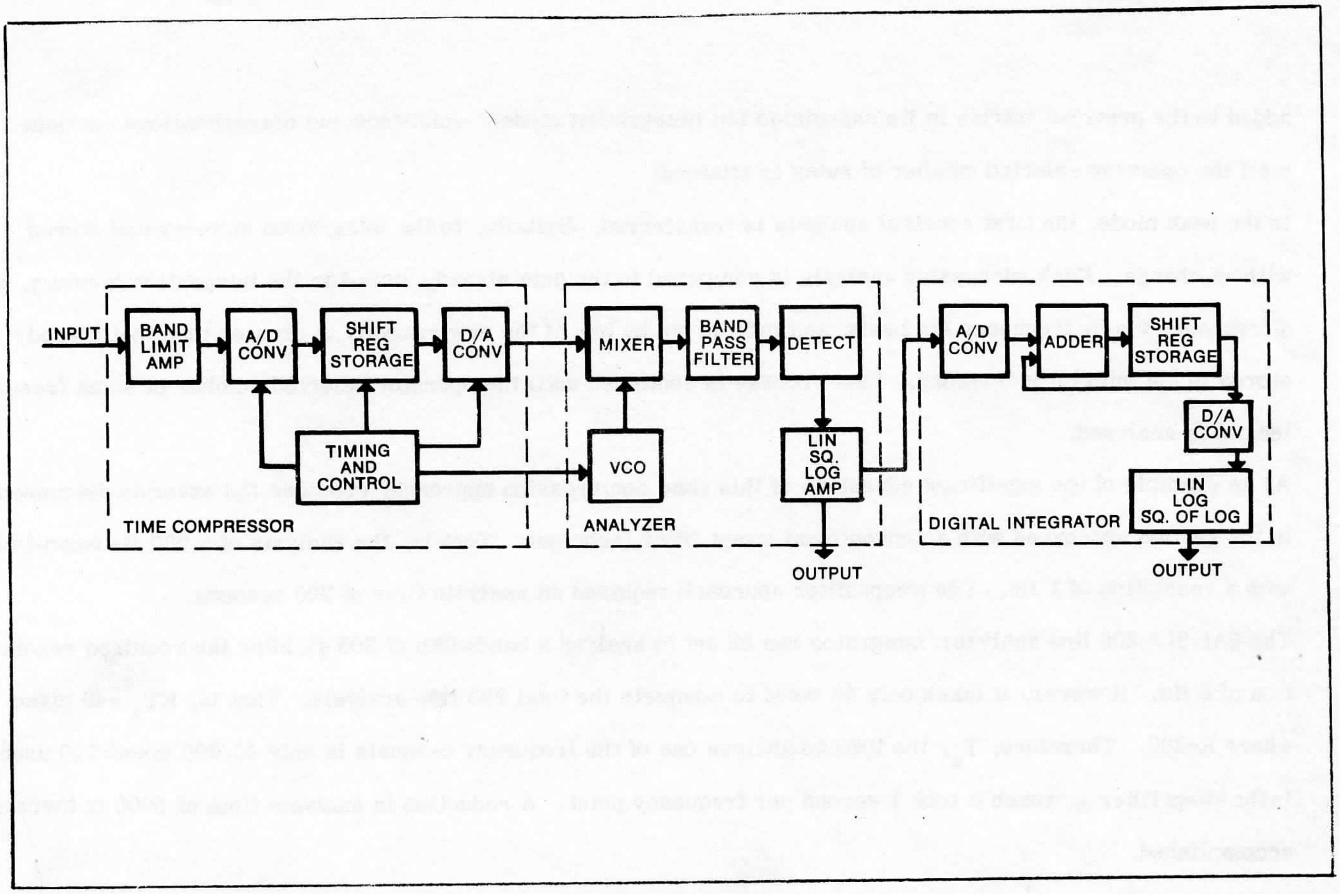

Figure 3.12: Block Diagram - Time Compression Analysis

The principle of operation of the SAICOR real time-time compression analyzer is as follows:

The input signal is band limited to a desired frequency range by a set of passive low pass filters (nine different ranges are available on standard units). This filtered signal is then converted to a series of digital samples at a rate consistent with the selected frequency range. These digital samples are stored in chronological order in a reliable drift free MOS FET shift register storage as shown in figure 3.12. The data length stored is automatically adjusted to be the reciprocal of the analysis bandwidth. The stored data is continually updated at the sampling rate selected and is read out of storage at a higher-than-acquisition rate. The ratio of the readout to acquisition rate corresponds to the time compression or speed-up ratio, and is the key factor in the Real-Time Analyzer operation. The sampling operation is represented diagramatically in Figure 3.11.

After conversion back to analog form, the stored data is mixed to the IF frequency, where it is "examined" by the narrow band pass filter. The detected output of the band pass filter represents the Fourier Analysis of the input function.

The analyzer output is available directly or can be applied to the digital integrator. In the latter case the analyzer output is converted to digital form and stored in the shift register storage of the integrator. This storage is divided into bins, each bin relating to one of the frequency resolution bins. Each new spectral coefficient is added to the previous entries in its associated bin (integration mode). Additions and normalizations continue until the operator selected number of sums is attained.

In the peak mode, the first spectral analysis is transferred, digitally, to the integration memory and stored without change. Each successive analysis is compared to the data already stored in the integration memory, on a frequency bin by frequency bin basis, andupdated bin by bin, if the new analysis is greater than that already stored in the integrator memory. This process is continued until the operator selected number of sums (record length) is analyzed.

As an example of the significant advantage of this time compression approach, consider the example discussed in the section concerned with a conventional swept filter technique. That is, the analysis of a 200 Hz bandwidth with a resolution of 1 Hz. The swept filter approach required an analysis time of 200 seconds.

The SAI-51A 200 line analyzer/integrator can be set to analyze a bandwidth of 200 yielding the required resolution of 1 Hz. However, it takes only 40 msec to complete the total 200 line analysis. That is, KT_a =40 msec where K=200, Therefore, T_a, the time to analyze one of the frequency channels is only 40/200 msec=200 usec. In the swept filter approach it took 1 second per frequency point. A reduction in analysis time of 5000 is thereby accomplished.

In the description of the principle of operation of the SAICOR real time analyzer several parameters need be selected. These include:

1. digitization and sampling rates - noted to be consistent with the frequency range selected.
2. processed signal length - noted to be dependent on the desired resolution.

The selection of these parameters and their interplay will be discussed in the next section.

3.3 PARAMETER SELECTION AND INTERPLAY

3.3.1 SAMPLING AND QUANTIZATION. The sampling theorem for low pass signals states that if a signal is constrained to lie in the frequency range from 0 to BHz, then this signal can be reconstructed (with zero error) by sampling the waveform at a rate of 2 B samples per secondand then properly processing these samples. That is, samples taken at the rate of 2 B samples/sec contain all the pertinent signal information. Figure 3.13 illustrates why this is so.

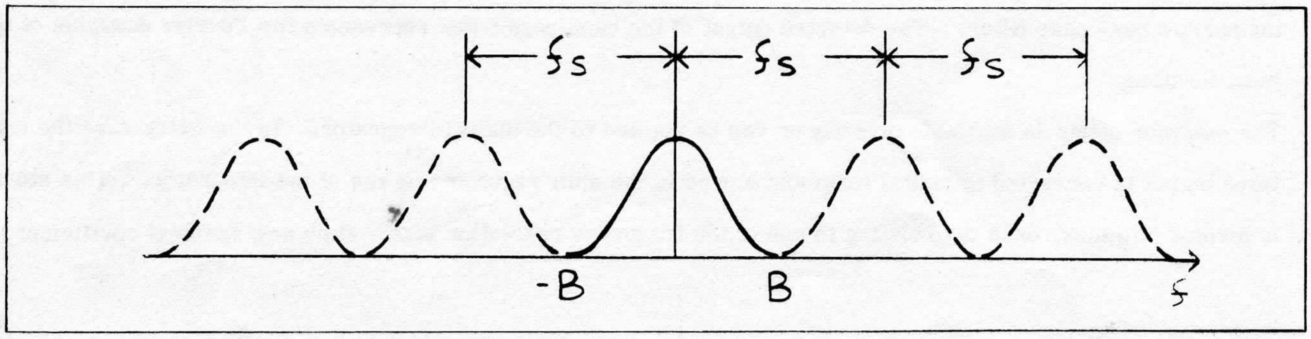

Figure 3.13: Frequency Aliasing due to Sampling

The solid curve represents the spectrum of the signal under consideration. The solid curve plus the dotted curve represents the spectrum of the sampled waveform. That is, the process of sampling produces additional spectral repeats shifted by multiples of the sampling rate f_s. These repeats or so-called "aliased" spectra must be shifted away from the original spectrum so that a low pass filter can then be used to eliminate them and yield, at the filter output, the original signal. As can be seen from this illustration, the minimum theoretical sampling rate required to accomplish this is f_s=2B samples per second. This implies that:

1. No spectral energy extends beyond BHz
2. An ideal low pass filter is available to completely pass all frequencies up to BHz and reject completely everything above BHz.

A more realistic situation is depicted in figure 3.14.

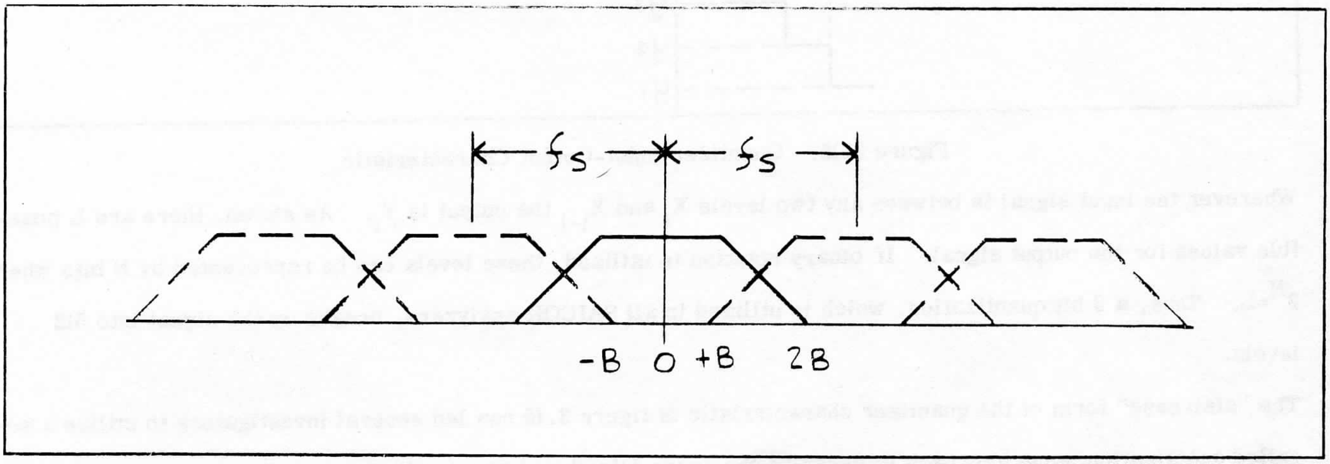

Figure 3.14: Considerations due to Finite Bandwidth

Here, the signal to be sampled has energy beyond BHz. To the extent that this is so, the sampling rate f_s must be increased so that the first spectral repeat does not produce distortion within the band from 0 to BHz. This distortion is often referred to as "foldover" or aliasing distortion. From figure 3.14 one notes that at a sampling rate of 3B samples/sec the foldover distortion depends on the amount of spectral energy present at 2B Hz. For example, if the signal energy is - 54dB at 2B, the foldover distortion will be less than -54dB within the range 0 to BHz. This is accomplished with a sampling rate of 3B.

The bandlimiting or so-called anti-aliasing front end low pass filters of the SAICOR real time spectrum analyzers are passive low pass filters which are flat (to $\pm$.1dB) within the range of 0 to BHz and are "down" greater than 60 dB at 2B. It is this sharp falloff characteristic which allows a 3B sampling rate (3 times the maximum frequency analyzed) to be used in the SAICOR instruments. This increase beyond the theoretical minimum (of 2B samples per second) is required to properly process the signal and obtain meaningful results.

In addition to sampling the analog waveform at the rate of 3B samples per second, these analog samples are to be converted to a digital word or representation. This implies a choice of the number of so-called bits-per word in binary notation or number of quantization levels. The input-output characteristic of the device which performs this digitization (quantizer) is shown in figure 3.15.

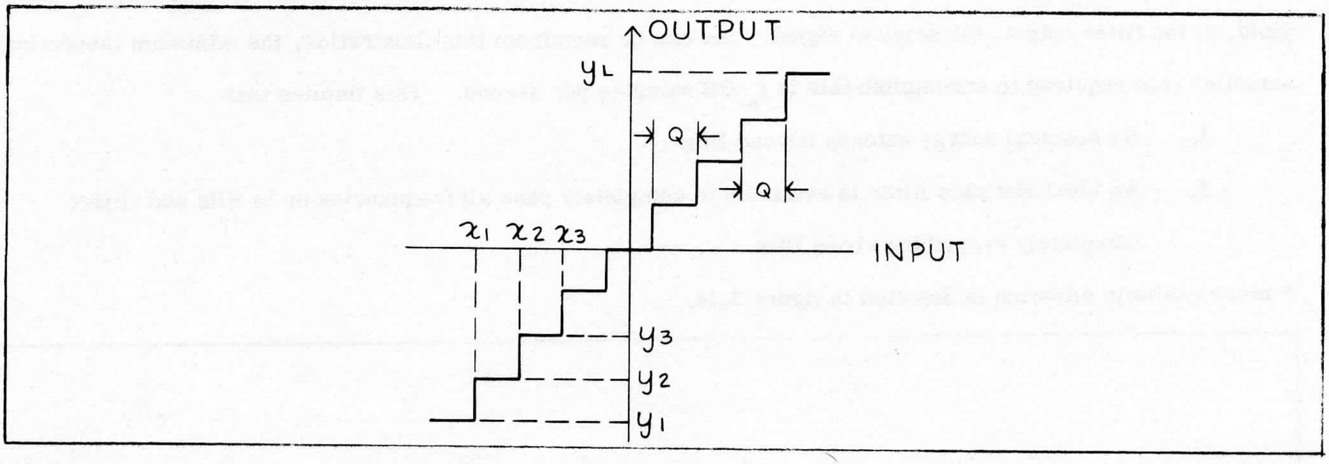

Figure 3.15: Quantizer Input-Output Characteristic

Wherever the input signal is between any two levels X_i and X_{i-1} the output is Y_i. As shown, there are L possible values for the output signal. If binary notation is utilized, these levels can be represented by N bits where $2^N = L$. Thus, a 9 bit quantization, which is utilized in all SAICOR analyzers, breaks up the signal into 512 levels.

The "staircase" form of the quantizer characteristic of figure 3.15 has led several investigators to utilize a so-called quantization noise approach to describe the error introduced by quantization. It has been shown that this quantization noise is related to the "step" size, Q, and therefore to the number of bits utilized. They have shown that for analysis purposes the quantizing effect can be approximated as an additive noise source as shown in figure 3.16.

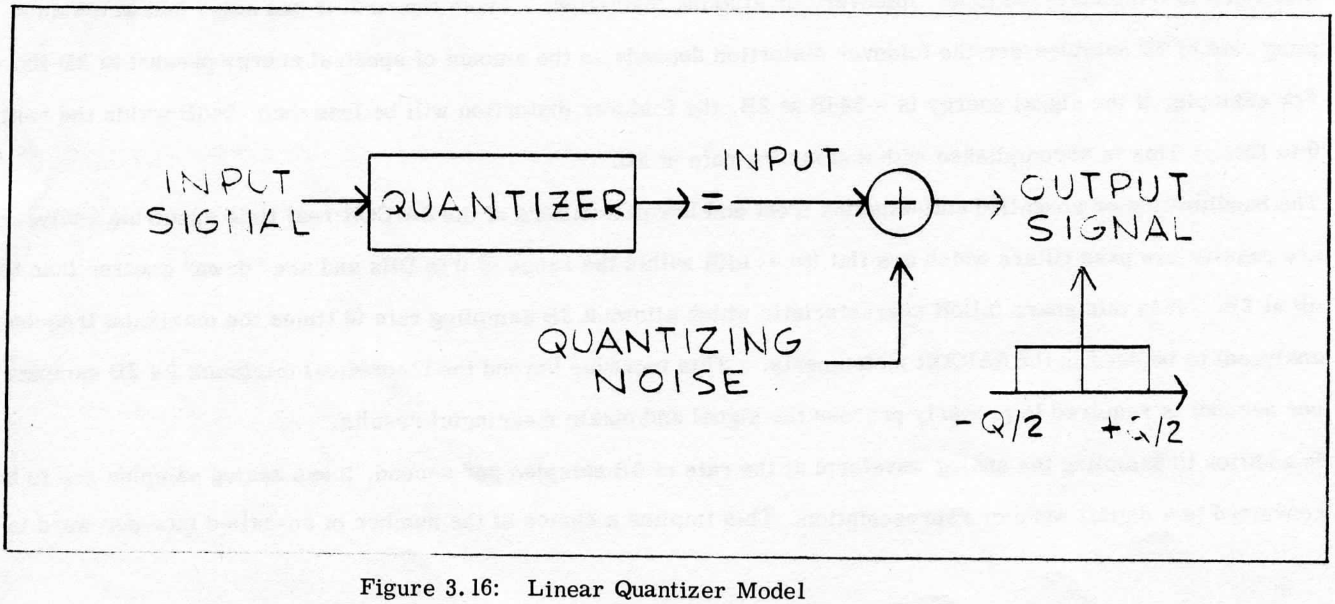

Figure 3.16: Linear Quantizer Model

This noise source can be shown to have a uniform probability distribution between $\pm Q/2$ where Q is the step size in the quantizer. From this representation it is shown that the signal-to-quantization noise increases by 6dB for every additional bit added. Moreover, the theoretical dynamic range capability is approximately equal to 6NdB where N is the number of bits utilized. Therefore, the SAICOR 9 bit analyzers have a 54dB dynamic range capability.

3.3.2 PROCESSED SIGNAL LENGTH AND TRUNCTION. In previous sections it was noted that the minimum signal length to be processed was reciprocally related to the desired resolution. The SAICOR analyzers process a signal which is continually being updated and is automatically adjusted to encompass ($1/B_R$) seconds of previous data where B_R is the resolution desired. Thus, Resolution=(1/Processed Signal Length). B_R is also defined as B/K where B is the maximum frequency or range selected and K is the number of lines in the analyzer. The following table summarizes these relationships.

FREQUENCY RANGE (Hz)	RESOLUTION 200 LINES (Hz)	PROCESSED SIGNAL LENGTH (200 LINES)	RESOLUTION 400 LINES (Hz)	PROCESSED SIGNAL LENGTH (400 LINES)
0-20	.1	10 sec	.05	20 sec
0-50	.25	4 sec	.125	8 sec
0-200	1	1 sec	.5	2 sec
500	2.5	.4 sec	1.25	.8 sec
1K	5	.2 sec	2.5	.4 sec
2K	10	.1 sec	5	.2 sec
5K	25	40 msec	12.5	80 msec
10K	50	20 msec	25	40 msec
20K	100	10 msec	50	20 msec

Note: Tabular data for the 400 line analyzer is included for comparison purposes only.

From the previous table it is noted that the processed signal length is finite in duration. The effect of processing this finite duration signal can be illustrated with figure 3.17.

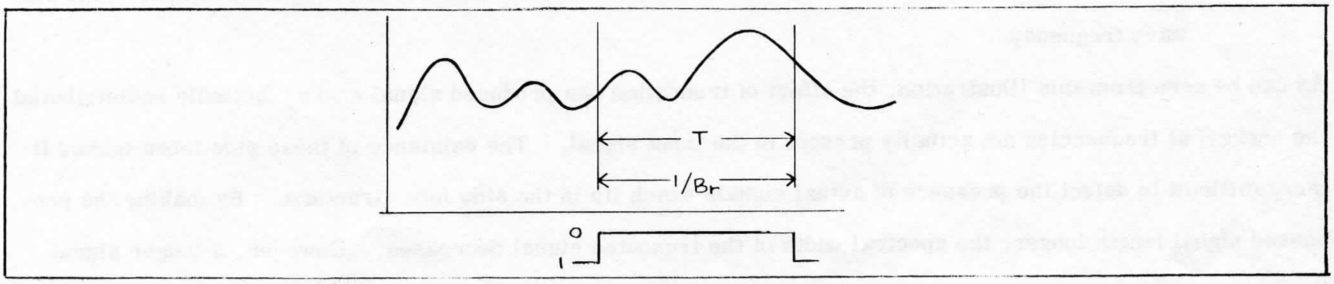

Figure 3.17: Finite Duration Window

The signal as shown extends to infinity but the analyzer, at any one time, is processing the signal within the rectangular "window" of duration $T=1/B_R$. The total effect of what the analyzer "sees" can be viewed as a multiplication of the rectangular window and the complete infinite length signal. This procedure truncates the original waveform. As might be expected this truncation procedure modifies the spectrum of the original signal. This modification can be shown to be a convolution effect in the frequency domain. This convolution effect, in general, broadens the original signal spectrum. This is illustrated in figure 3.18 for an original signal which is an infinite length sine wave.

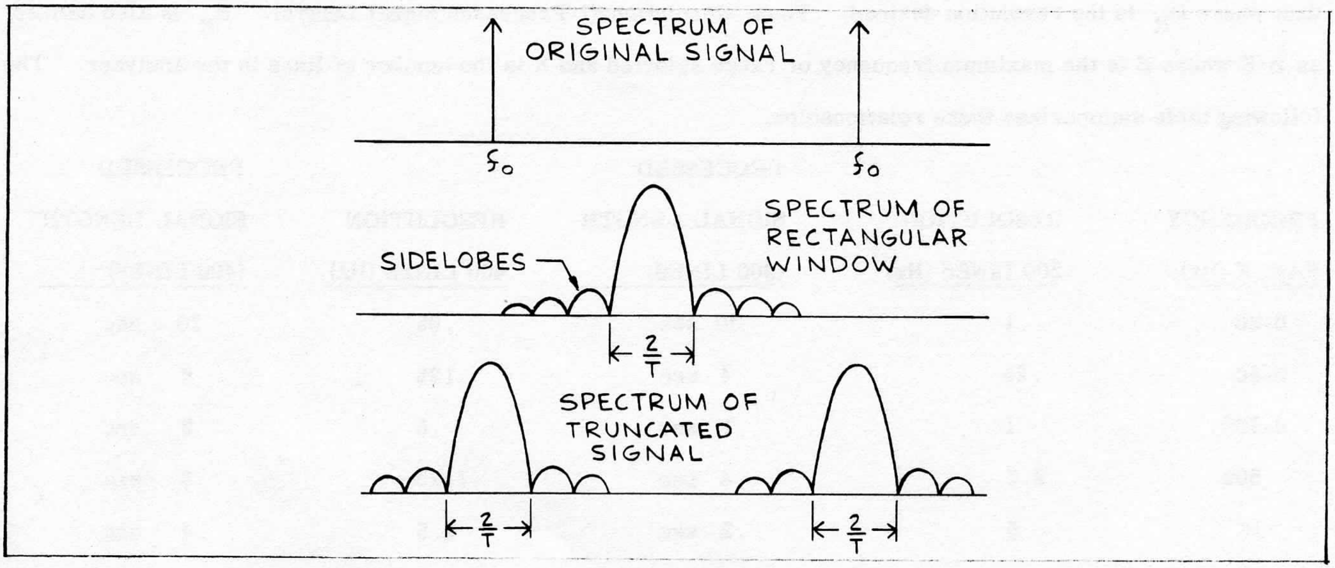

Figure 3.18: Effects of Convolution

Several items are to be noted:

1. The original infinite length sine wave had a "zero width" spectrum (a delta function).
2. The width of the spectrum of the window is inversely related to the width of the rectangular window or processed signal duration.
3. The spectrum of the window has side lobes which decay rather slowly (according to a sine x/x relationship) or (sine x) ÷ x.
4. The spectrum of the truncated signal has broadened from that of a delta function centered at the sine wave frequency.

As can be seen from this illustration, the effect of truncation has produced signal energy (actually redistributed the energy) at frequencies not actually present in the input signal. The existance of these side lobes makes it more difficult to detect the presence of actual signals which lie in the side lobe structure. By making the processed signal length longer, the spectral width of the truncated signal decreases. However, a longer signal length dictates increased memory size in the analyzer. By changing the nature of the window function, the shape of the spectrum can be altered so that the side lobes decay more rapidly than the (sine x)/x falloff associated with a rectangular window. It should be noted, however, that a price must be paid for this sidelobe suppression. This price is paid in terms of an increase in the 3dB and noise bandwidth associated (the noise bandwidth of a filter is defined as the width of an ideal rectangular filter which would pass the same amount of power as the actual filter when broadband noise is applied to its input) with the spectrum of the window function. This increase in width decreases the attainable resolution of the analyzer to a number which is greater than the reciprocal of the processed signal length. The concept behind the use of all window functions is associated with the fact that sharp truncations in the time domain (as with the rectangular function) cause slow decay of the spectral lobes in the frequency domain. A multitude of useful window functions have been used in the past. These include:

1. cosine
2. $(\text{cosine})^2$ or Hanning weighting
3. $(\text{cosine})^N$ for higher powers of N
4. $(\text{cosine})^2$ on a 8% pedestal or Hamming weighting
5. $(\text{cosine})^2$ times an exponential or Triplett weighting
6. and even the use of such elegant functions as prolate spheroidal waveforms.

In short, there is no best weighting function for all criteria. Three items which should be considered in choosing a weighting function are:

1. The 3dB bandwidth
2. The noise bandwidth
3. The sidelobe level

Ideally the 3dB and noise bandwidth should not increase - their ratio should be 1-(as with an ideal filter) and there should be no side lobe levels. Since no weighting function will satisfy all of these requirements, compromises must be made. The weighting function included in the SAICOR spectrum analyzers is the Hamming weighting or $(\text{cosine})^2$ on a 8% pedestal (available via a front panel switch). This represents the best compromise for spectrum analysis in that the side lobe level is reduced from -13dB (for the rectangular function) to -42dB and the ratio of noise bandwidth to 3dB bandwidth is only 1.04.

Although the Triplett weighting has no sidelobes (ie. a monotonic decrease in the spectrum - which is rather slow) its 3dB bandwidth is 15% larger than the Hamming; its noise bandwidth is 20% larger and the ratio of noise is 3dB bandwidth is 1.07 which is also larger than the Hamming function.

It should also be noted that the noise bandwidth associated with the weighting function utilized is not the noise bandwidth of the spectrum analyzer. In addition to accounting for the weighting function, the analysis filter when used in its swept mode must also be considered. These factors will be discussed in section 4 of this manual.

3.3.3 MEMORY SIZE. Up to this point several parameters for the analyzer have been fixed. These include:

1. The sampling rate, which is 3B samples/sec. B is the bandwidth selected (0 to BHz) or so-called maximum frequency component.
2. The processed data length which is $T=1/B_R$ where B_R is referred to as the analyzer resolution.
3. The number of lines or frequency channels in the analyzer which is $K=B/B_R$.
4. The number of bits (per sample) used for quantizing the input which is 9 (or 512 levels).

These parameters allow the determination of the size of the digital MOS FET memory used in the SAICOR analyzers as follows. In a time T, the number of samples taken is $3Bt=3B/B_R$. But $K=B/B_R$. Therefore, the memory size is 3K words or-three times the number of lines in the analyzer-times 9 bits per word. Thus there is a 600 word memory in the 200 line analyzers (SAI-51ASAI-53A) and a 1200 word memory in the 400 line analyzers (SAI-52A SAI-54A).

3.4 USE OF A DIGITAL INTEGRATOR

3.4.1 AVERAGING FOR IMPROVING ESTIMATES. In section 3.2.1 a model was described which can be used to determine the power in a bandwidth B_R Hz. This model is shown again in figure 3.19.

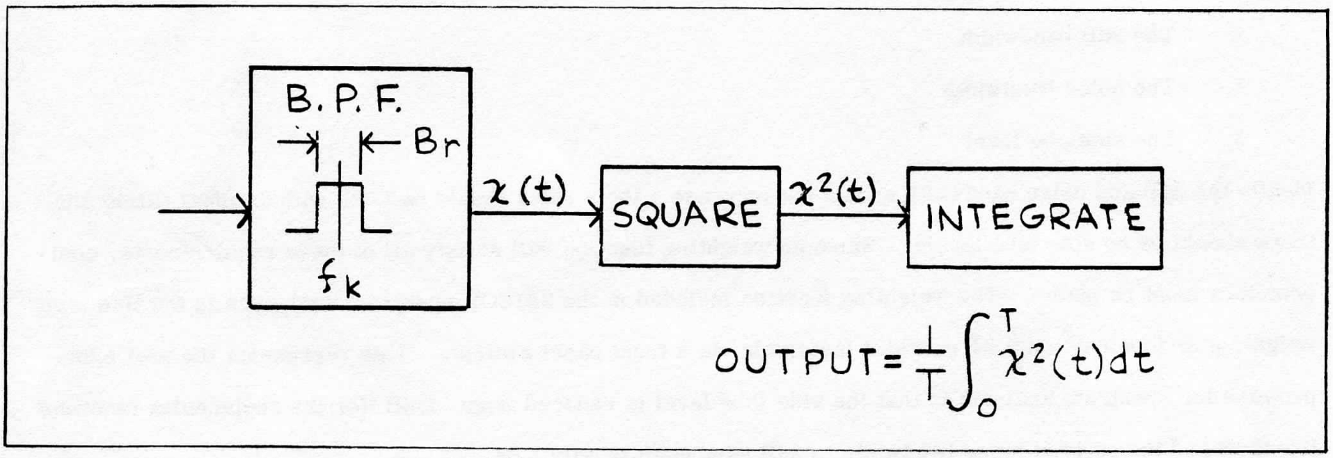

Figure 3.19: Power Measurements

The discussion in section 3.2.1 also noted that the integrator can be thought of as adding independent numbers at the rate of B_R per second. With this in mind, the integrator output of figure 3.19 can be written as

$$\text{Output} = \frac{1}{B_r T} \sum_{i=1}^{B_r T} x_i^2 \qquad (1)$$

The real time spectrum analyzer was shown to process signals with a sliding window that is $(1/B_R)$ seconds in length. Therefore, if the total time T is expressed as N times $(1/B_R)$ the output becomes

$$\text{Output} = \frac{1}{N} \sum_{i=1}^{N} x_i^2 \qquad (2)$$

In this form, the X_i components are available at the analyzer output and the summation of N components can be accomplished in a digital integrator or averager. The question is, why average ? The answer, for periodic signals, is that there is no need to because the successive numbers X_i are identical from analysis to analysis. However, for random signals such as exists in machinery noise problems, vibration and acoustics and many sonar, radar and communication problems, the number X_i represent only an estimate of the power in a band. As with most estimates, the longer the observation interval, the better the estimate becomes. In order to determine the relationship between the improvement in the estimate and the number of summations consider figure 3.20.

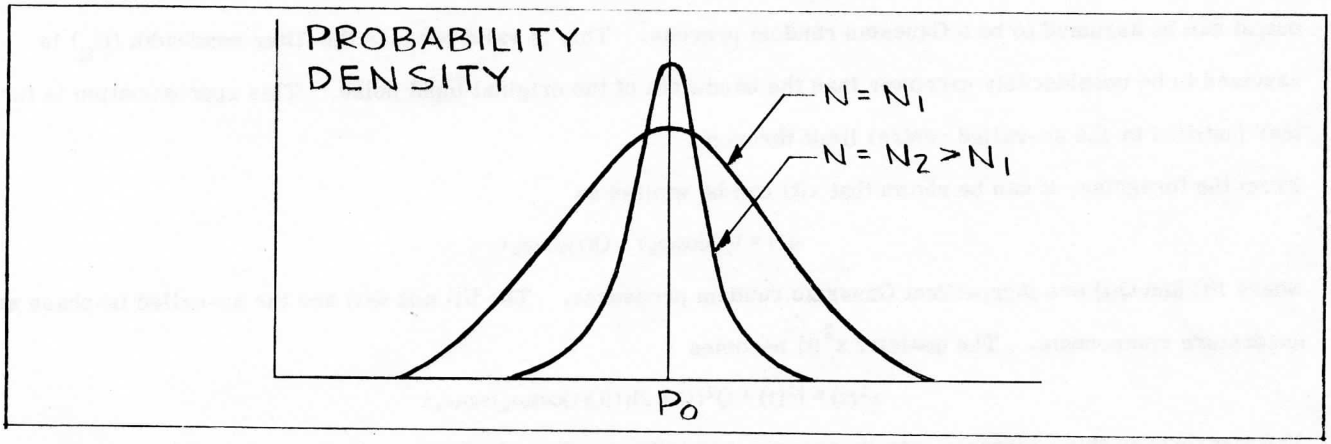

Figure 3.20: Improved Estimation due to Averaging

When N is infinite, representing an infinite averaging time, the "estimate" given in equation (2) is identical to the true power P_o within the band B_R. However, for finite values of N the curves indicate that values other than P_o can be obtained. As N increases the curves become more and more peaked about P_o, the true value. The curves shown are so called probability density curves and have a unity area. The area within a certain % around P_o yields information concerning confidence regions as shown in figure 3.21.

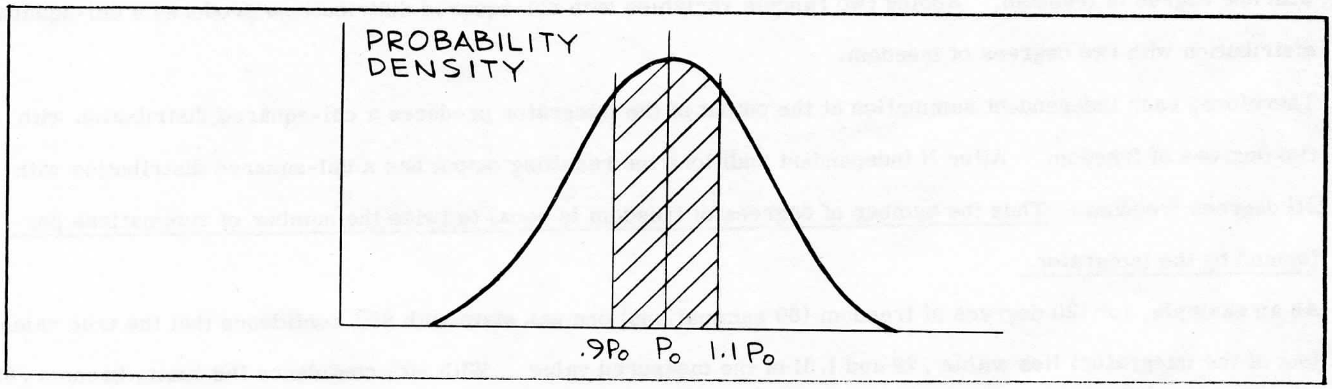

Figure 3.21: Determining Confidence Regions

As an example, verticle lines are drawn at $\pm$ 10% about P_o. The shaded area represents the "confidence" one has that the answer out of the integrator will be within $\pm$ 10% of the true answer. If this area was equal to .45, than with 45% confidence the answer will be correct to within $\pm$ 10%. For larger values of N the confidence might go up to 90% for the same $\pm$ 10% region.

In order to obtain specific numbers, the shape of the curves of figure 3.21 must be obtained. This has been done and the results tabulated in many statistics books. The framework for the calculation is as follows: When random noise is applied to the input to the bandpass filter (which is identical to the input to the analyzer) the filter output can be assumed to be a Gaussian random process. This is valid because the filter bandwidth (B_R) is assumed to be considerably narrower than the bandwidth of the original input noise. This approximation is further justified by the so-called central limit theorem.

From the foregoing, it can be shown that x(t) can be written as

$$x(t) = I(t)\cos\omega_k t + Q(t)\sin\omega_k t$$

where I(t) and Q(t) are independent Gaussian random processes. The I(t) and Q(t) are the so-called in-phase and quadrature components. The quantity $x^2(t)$ becomes

$$x^2(t) = I^2(t) + Q^2(t) + 2I(t)Q(t)\cos\omega_k t\sin\omega_k t$$

The last term in this expression can be shown not to influence the calculations and the output of the averager then becomes

$$\text{Output} = \frac{1}{N}\sum_{i=1}^{N} x_i^2 = \frac{1}{N}\sum_{i=1} (I_i^2 + Q_i^2)$$

It can be shown that the statistical properties $I_i^2 + Q_i^2$ can be described by a chi-squared distribution in 2 degrees of freedom. In essence, squaring a Gaussian random variable (I or Q) produces a chi-squared distribution with one degree of freedom. Adding two random variables with chi-squared distributions produces a chi-squared distribution with two degrees of freedom.

Therefore, each independent summation at the output of the integrator produces a chi-squared distribution with two degrees of freedom. After N independent additions the resulting output has a chi-squared distribution with 2N degrees freedom. Thus the number of degrees of freedom is equal to twice the number of summations performed by the integrator.

As an example, for 120 degrees of freedom (60 summations) one can state with 95% confidence that the true value (out of the integrator) lies within .79 and 1.31 of the measured value. With 80% confidence the limits become .85 and 1.19 of the measured value. Detailed tabulations can be found in a publication written by R. A. Fisher. (Statistical methods for Research Workers, Edinburgh and London 1941).

3.4.2 NON REDUNDANT AVERAGING. The previous discussion assumed that the summations were accomplished with independent measurements. This concept of independence implies that the measurements have been made on non-overlapping or so-called non-redundant data.

Consider the 200 line SAI-51A operating on the 5 kHz scale. The resolution on this scale is B_R=25Hz and, therefore, the processed signal length is 40 msec ($1/B_R$). The analysis time (to complete a full 200 line analysis) is also 40 msec. Therefore, successive measurements of the analyzer are made on nonredundant data. However, on the 1KHz scale the processed record length is 200 msec ($1B_R$=1/5Hz) but the analysis time is still 40 msec. This means only every fifth spectrum analysis is accomplished on nonredundant data. The averager in the SAICOR analyzer automatically will integrate only every fifth spectral output. Therefore, the integrator is performing nonredundant averaging. This procedure means that the confidence levels for the same number of summations on both scales in the example are the same.

3.4.3 SPECTRAL COMPARISON. In addition to the conventional integration or averaging mode built into the SAICOR analyzer/averager, a PEAK mode is included. The function of this mode is to allow a scan-to-scan spectral comparison while retaining the maximum or "peak" energy in each bin throughout the total time interval selected. Its operation is as follows:

Spectrum Analysis #1 is performed and the results stored in the integrator. Analysis #2 is then performed and the resulting amplitudes are compared on a line by line frequency basis with the integrator retaining the larger of the two numbers in each of the 200 frequency channels. Analysis #3 is then performed and compared with the previous result. This procedure continues for the preselected number of record lengths as indicated by the SUMS PER BIN switch on the instrument. The resulting answer represents the peak spectral content of the signal over the entire time interval selected. This mode is extremely useful in shock analysis in that conventional averaging would suppress high energy short bursts when they become averaged with the low level spectral amplitudes. The peak mode is also useful in automatic swept sine wave transfer function analysis.

3.5 CONCLUSION

The theory of operation has been discussed in this section and now the remainder of the manual will be concerned with operation and function of all controls as well as signal flow and calibration details.