

**Subject:** Re: Geometric projection help request

**From:** Dave Typinski <davetyp@typnet.net>

**Date:** 09/04/2015 18:39

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Hi All,

For those who are interested but haven't followed the thread over on the SARA list, the steered beam shape problem has been solved thanks in large part to Francisco, who helped me with a very good paper on the subject.

Turns out that if you plot a phased array's beam shape in "direction cosine space" -- a polar view that's an orthographic projection of the visible sky hemisphere down onto a flat plane -- then the beam shape does not change with beam steering.

However, since we don't live in cosine space, but in regular old Newtonian linear space (near enough), we have to convert that to something more useful. The deformations in the pattern due to beam steering become evident when one translates the cosine space plot into a common AZ/EL polar plot -- which has a linear radial scale instead of a cosine radial scale.

At least that's how it works if you have isotropic radiators for array elements. The beam shape change due to a non-isotropic element pattern is a separate deformation, one which was addressed in one of the Green Bank presentations last year.

The challenge now is to figure out how to combine both deformations mathematically to predict the expected real-world beam shapes of the TFD array.

Here are some neat pictures, for those of you interested in such stuff. Those who aren't are encouraged to use it as a sleep aid if your eyes haven't glazed over already.

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Dave

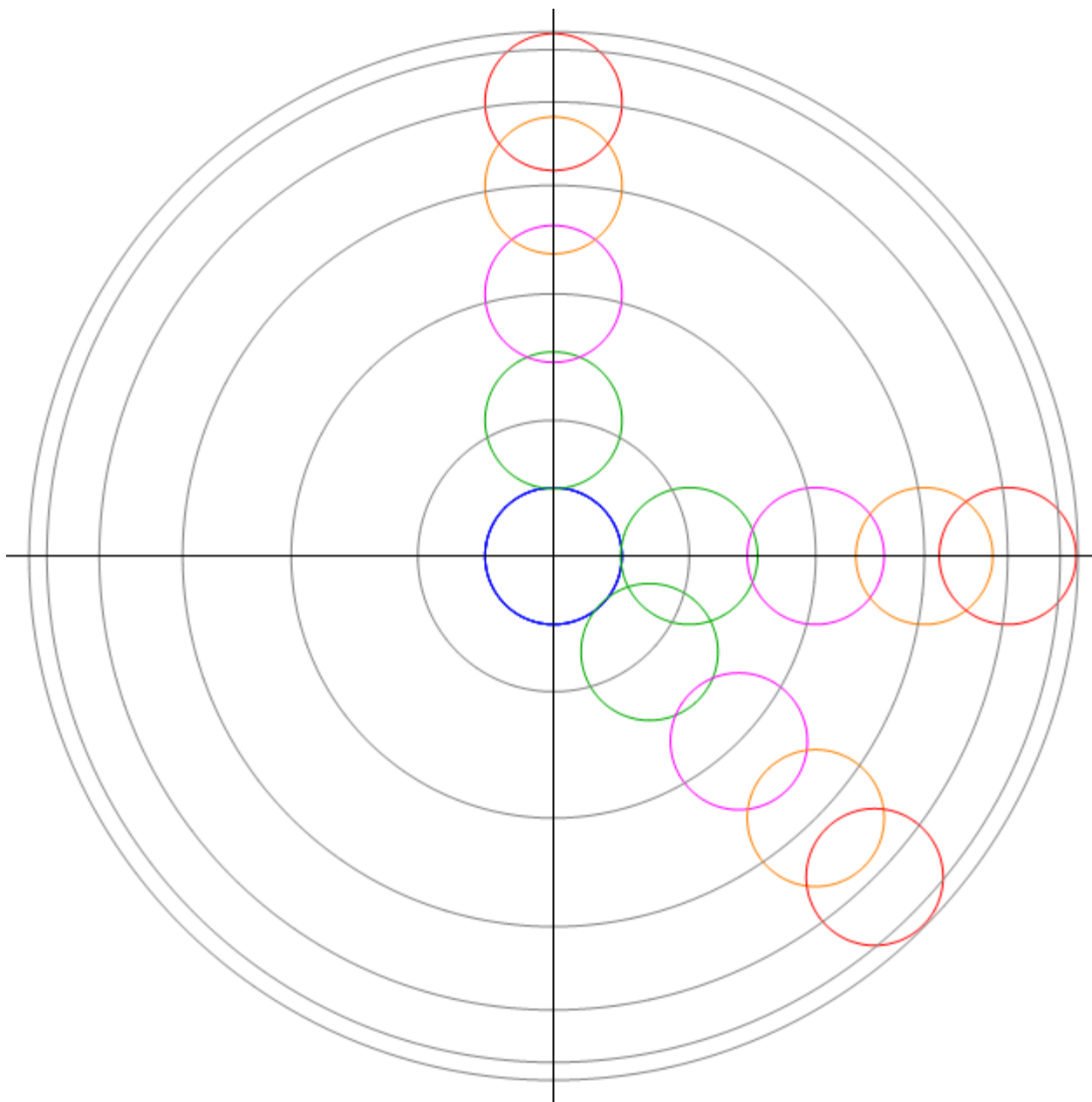
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**15 deg circular HPBW**

**Direction cosine space (otherwise known as an orthographic projection of the sky dome)**

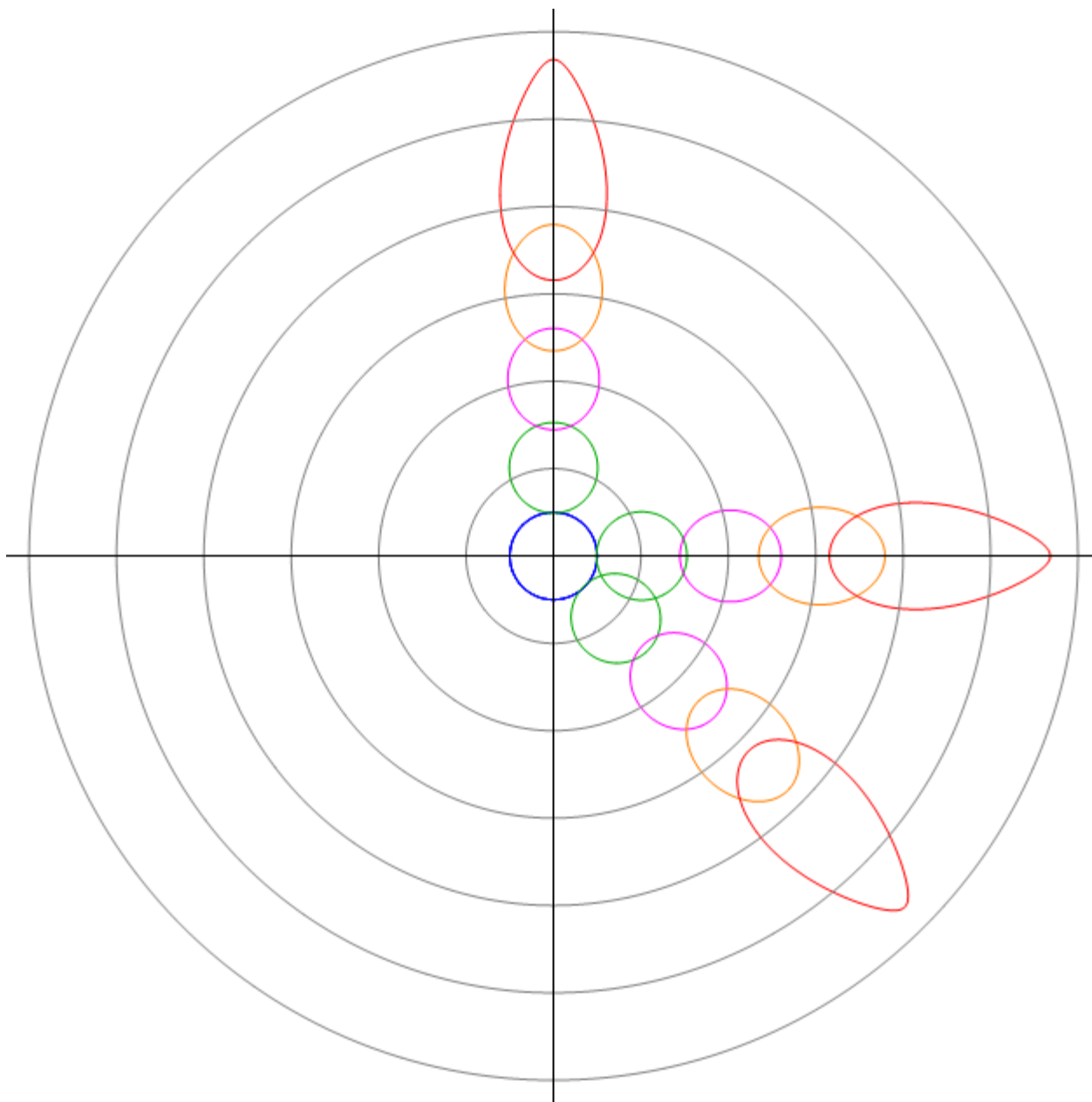
**15 deg radial grid spacing**

**15 deg beam elevation increments, 30 degs to 90 degs**



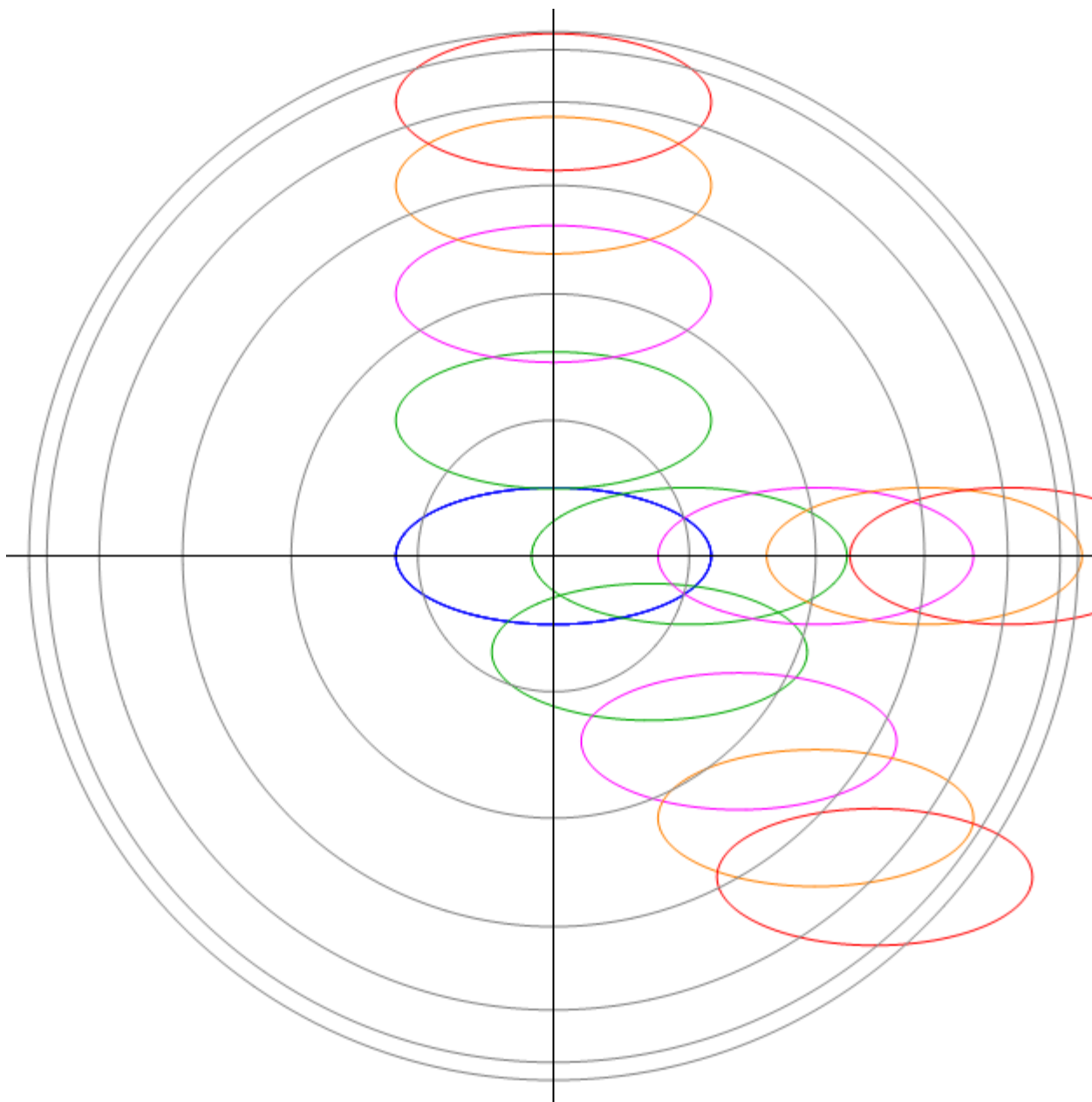
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**15 deg HPBW**  
**polar AZ/EL sky map**  
**15 deg radial grid spacing**  
**15 deg beam elevation increments, 30 degs to 90 degs**



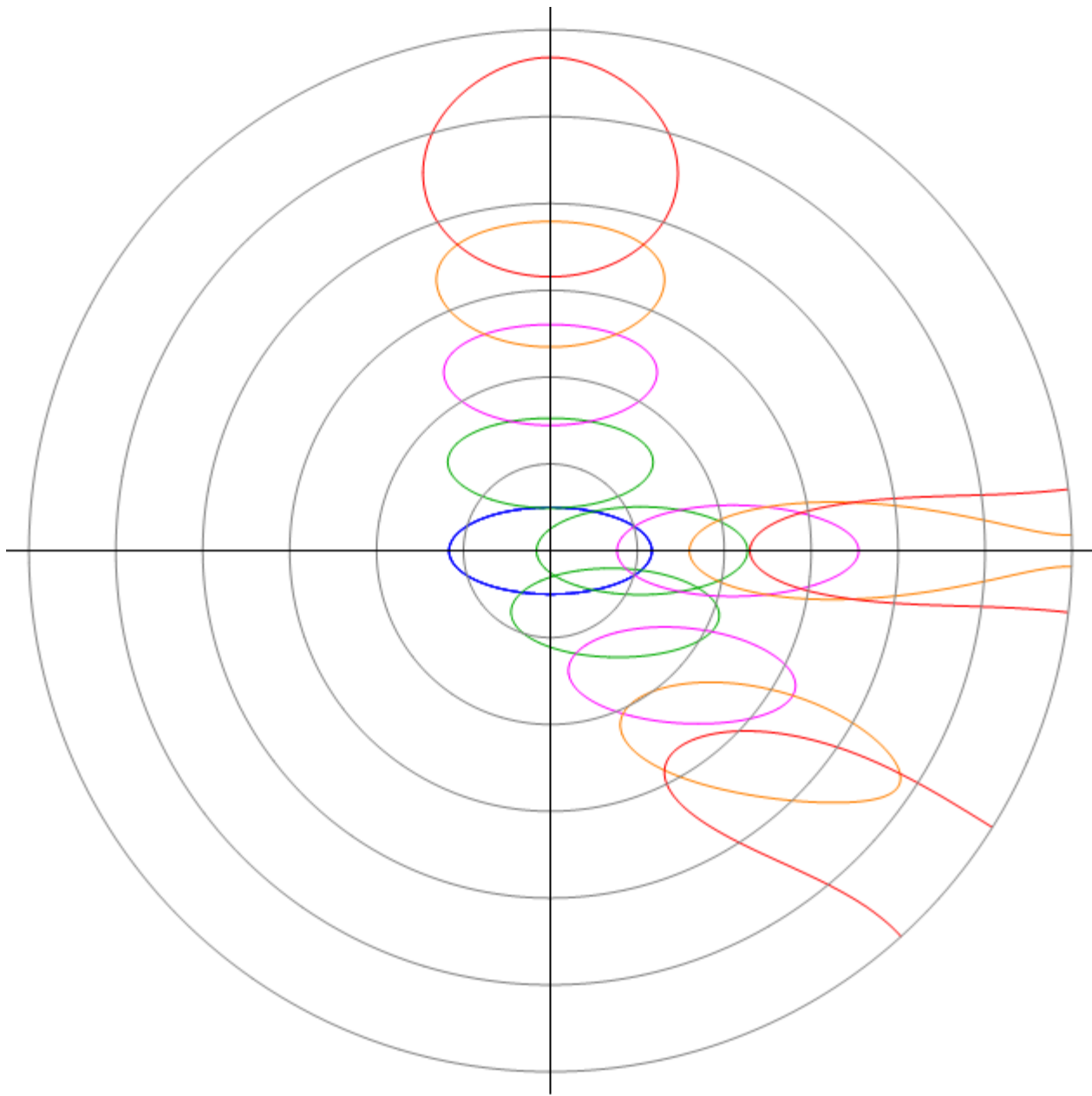
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**TFD Array 20 MHz HPBW 15x35 degs**  
**Direction cosine space**  
**15 deg radial grid spacing**  
**15 deg beam elevation increments, 30 degs to 90 degs**



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**TFD Array 20 MHz HPBW 15x35 degs**  
**Polar AZ/EL sky map**  
**15 deg radial grid spacing**  
**15 deg beam elevation increments, 30 degs to 90 degs**



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On 09/01/2015 18:53, Dave Typinski wrote:

I wonder if anyone here can point me in the right direction for a geometric projection problem.

Here's the deal.

**Fact 1:**

When you scan the beam of a phased array radar (or radio telescope) across the sky, the beam changes shape the further you go from the direction normal to the plane of the phased array.

E.g., if the antenna array is planar and horizontal, and if it produces a circular beam when pointed at zenith, then any beam steering progressively changes the beam shape -- and, the closer the beam gets to the horizon, the worse the deformation.

This is shown very well in the figure below, which is from Skolnik's excellent book, "Introduction to Radar Systems".

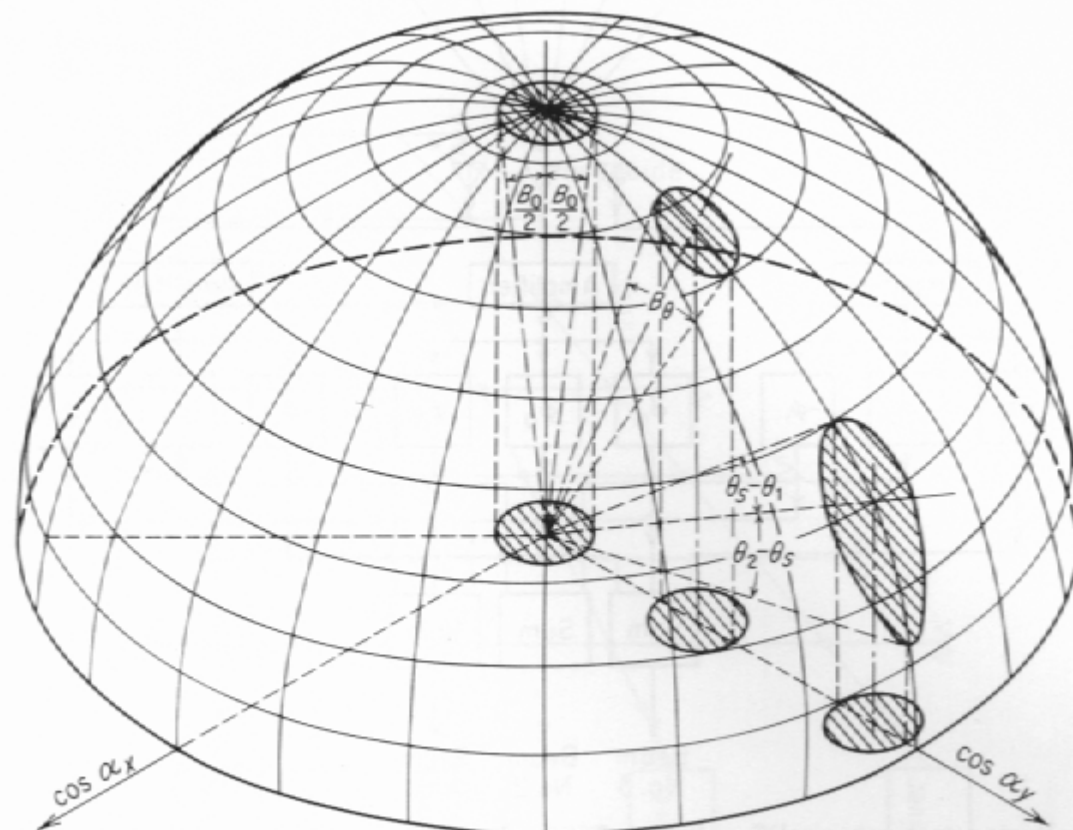
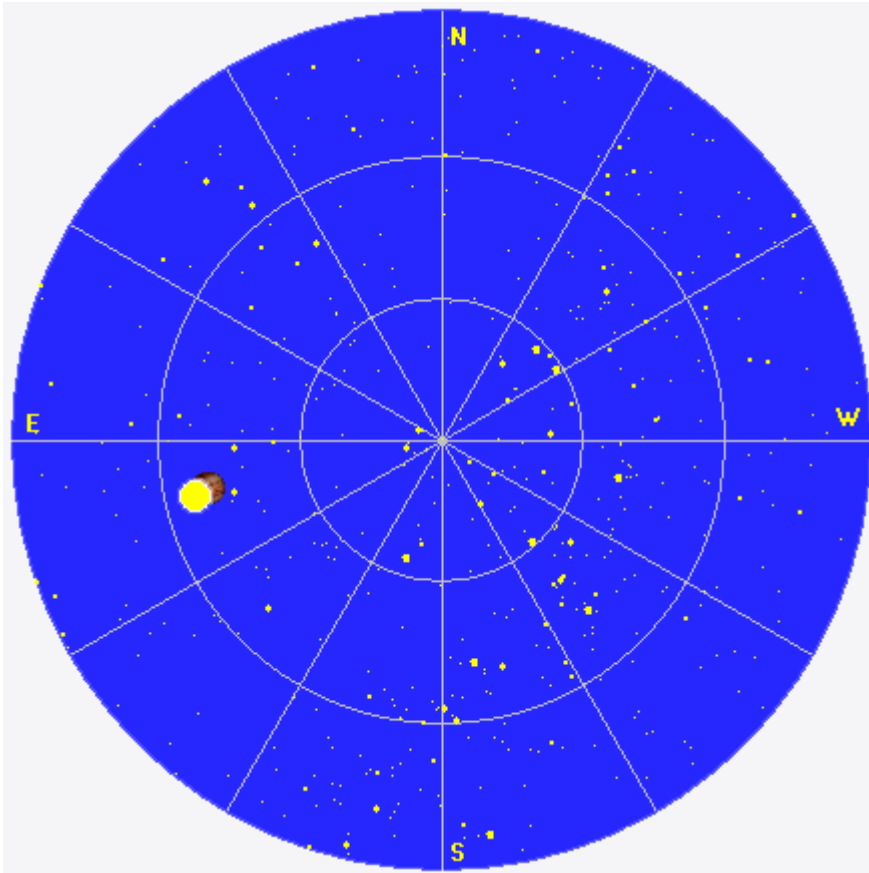


FIG. 7.36. Beamwidth and eccentricity of the scanned beam. (From Von Aulock,<sup>70</sup> courtesy Proc IRE.)

### Fact 2:

Radio Jupiter Pro shows the position of objects in the sky on a polar plot. An example is shown below:

Jupiter and the Sun are shown off to the east, real close together. The concentric rings are 30 degs of elevation, with zenith at the center of the image.



**The problem:**

How can one correctly plot that teardrop-shaped beam (shown in the radar illustration) onto the polar coordinate sky map?

At first, it may seem like this is simple, that when the beam is projected onto a plane, it's a circle. But this is not so.

It would only be so if the polar sky map did not have even divisions in the radial direction. If the rings of elevation were spaced according to a cosine curve, then the beam shape would be a circle. But, the spacing is linear. So it's not a circle in polar coordinates.

But if not a circle, then what shape /is/ it in polar coordinates?

How does one translate the plane shown in the radar diagram (showing three circles) to the plane shown in the sky map?

Any ideas?

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Dave