Subject: Re: [SARA] Geometric projection help request
From: Dave Typinski <davetyp@typnet.net>
Date: 09/04/2015 11:30
To: sara-list@googlegroups.com

HI Jim,

I see now, the 1/cos doesn't actually do the transform, it just provides the broadening of the beam relative to the HPBW at zenith, assuming zenith is normal to the plane of the array. Makes sense, and I see where Skolnik mentions it. Thanks for the pointer.

Some plots below. Made them using the polar form of the equation for an ellipse, scaling the radial axis of the polar plot to put it in direction cosine space (the space in which a circular beam stays circular when steered off zenith), then rescaled the radial axis to linear to make the AZ/EL plots (used arcsin).

I note that the pear-shaped beams in your plots are rotated about the beam centerline 180 degs to these. Skolnik and Von Aulock show the fat part of the pear-shape at the higher-elevation end of the beam pattern when the beam is aimed down toward the horizon.

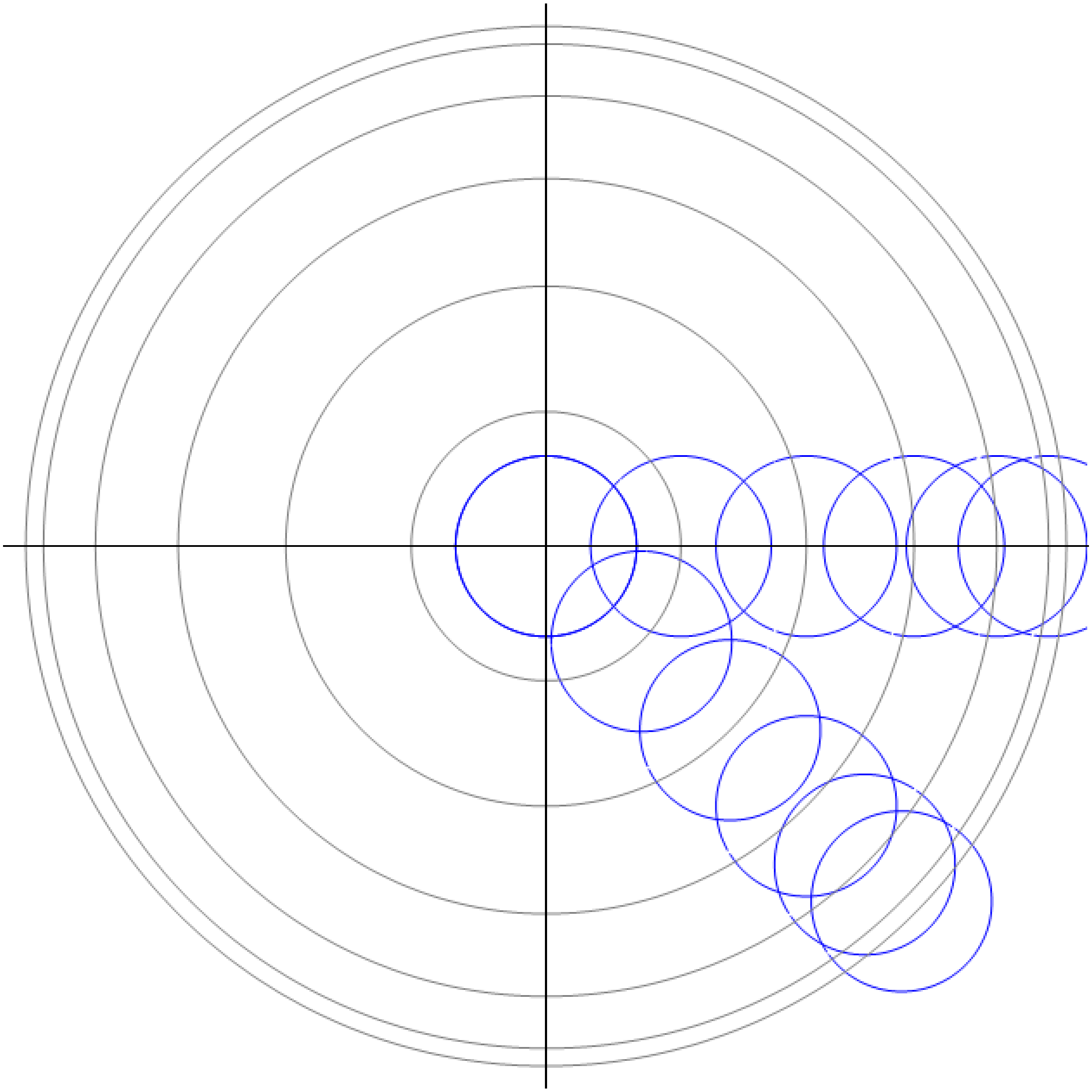
Also, the 20 degs HPBW at AZ45 EL30 beams in our plots show some wild differences in size. Mine goes off the edge of the sky, but yours does not.

Thus, I have to wonder if one can accurately plot the beam shape by using just the 1/cos broadening relationship.

The plots for the TFD array (the oval beams shown below) still aren't the real beam. They do not yet include the effect of the element pattern, only the distortion due to steering alone.
--
Dave

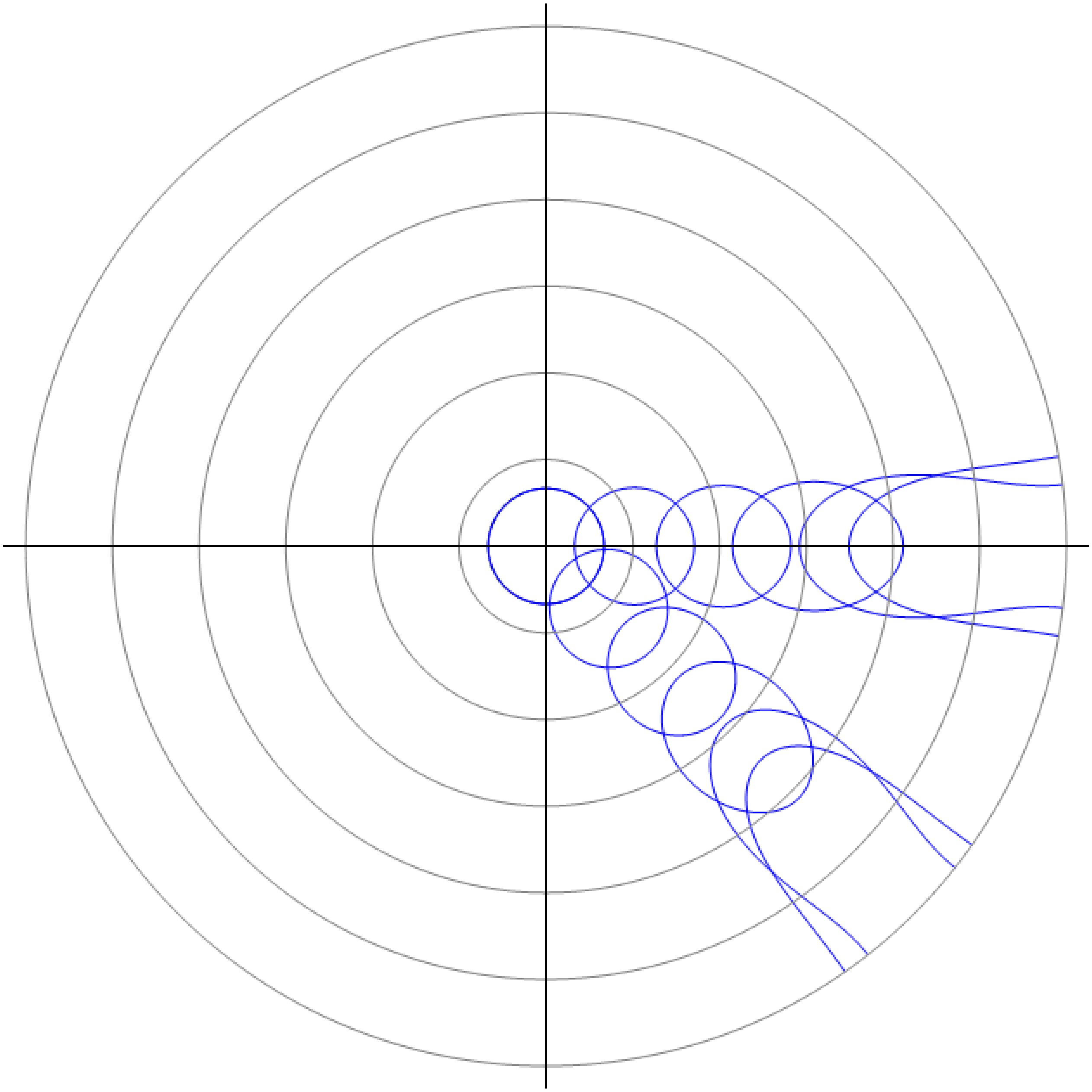
---

20 deg HPBW, direction cosine space (otherwise known as an orthographic projection of the sky dome), 15 deg radial grid spacing and 15 deg beam elevation increments

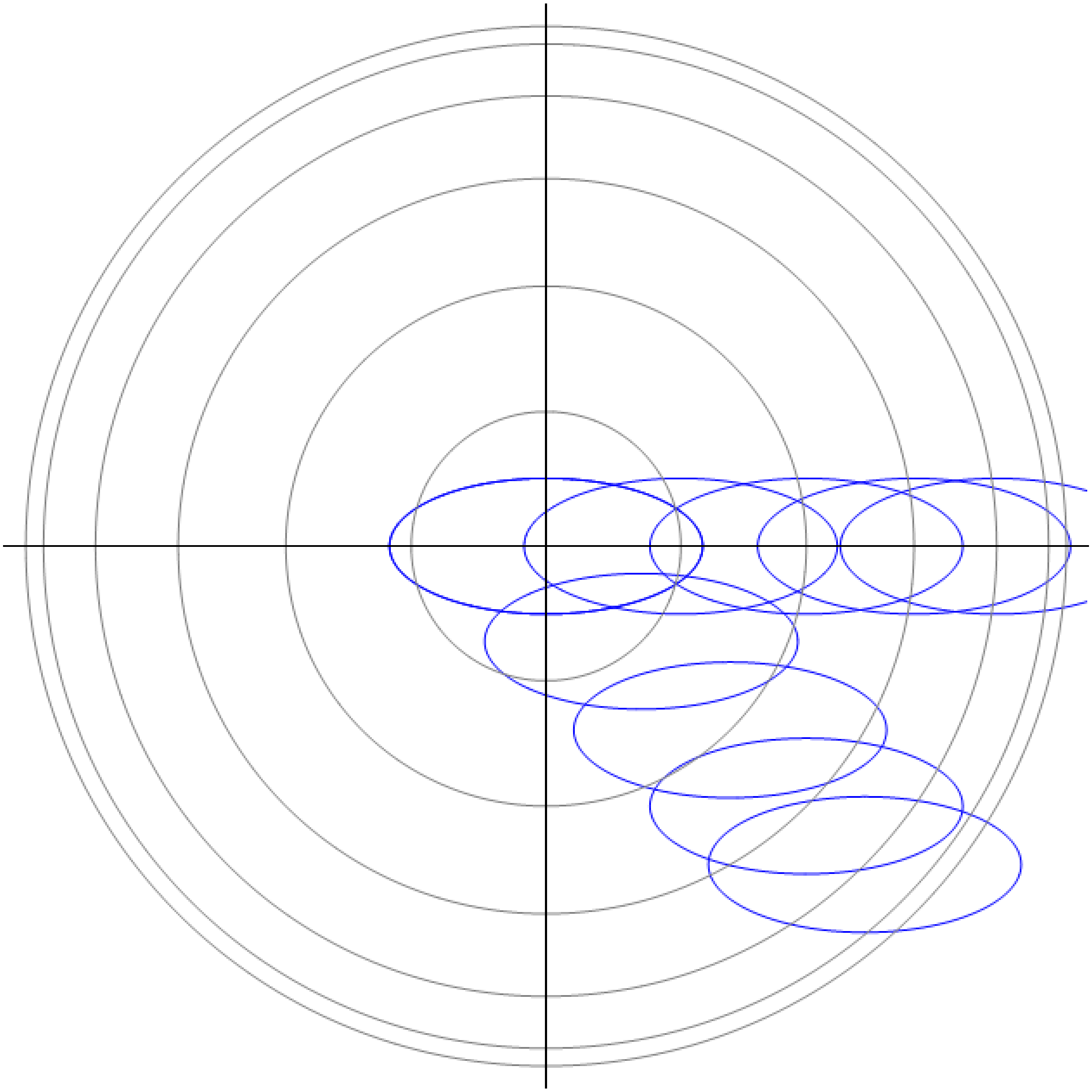


---

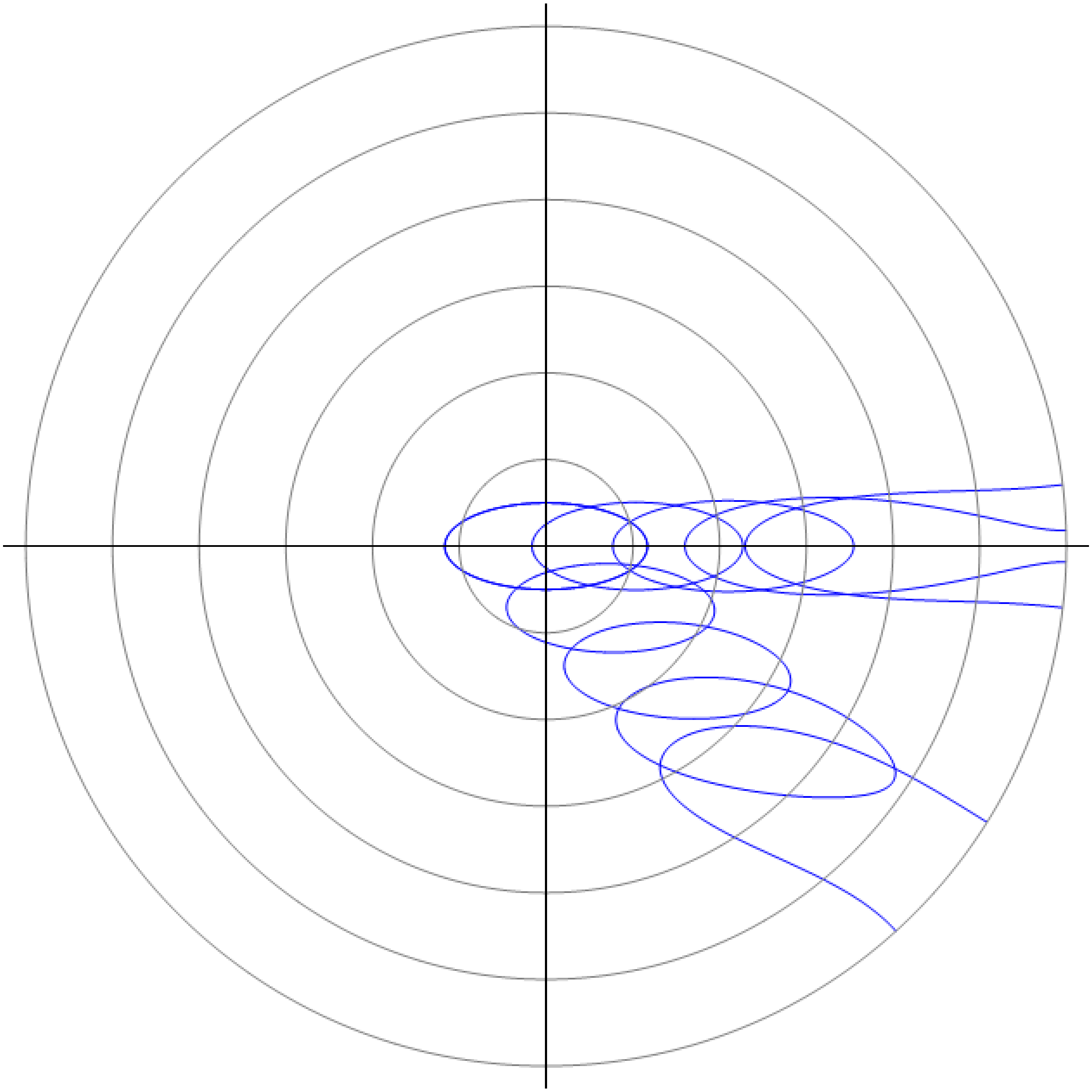
20 deg HPBW, polar AZ/EL sky map, 15 deg radial grid spacing and 15 deg beam elevation increments



TFD Array 20 MHz HPBW 15x35 degs, direction cosine space, 15 deg radial grid spacing and 15 deg beam elevation increments



TFD Array 20 MHz HPBW 15x35 degs, polar AZ/EL sky map, 15 deg radial grid spacing and 15 deg beam elevation increments



On 09/03/2015 21:45, James Abshier wrote:

> Hi Dave,
>
> First of all I should have said 1/cos(zenith angle) or 1/sin(elevation). (my bad)
>
> If you consider the projection of the array on a plane normal to the squinted beam, its vertical extent in the normal plane will be reduced by cos(zenith angle). At zenith, the projection will be full size, and as the beam is squinted, the array projection will be reduced in vertical extent (on the normal plane). The Fourier Transform of the array (the beam pattern) will then be broadened by 1/cos(zenith angle) in the vertical dimension (approximately). In my code, I used 1/sin(elevation) which is equal to 1/cos(zenith angle).

Consider the standard formula for half power beamwidth of a circular aperture with uniform illumination:

Half Power Beamwidth = 58 degrees / Diameter in wavelengths

In other words there is an inverse relationship between aperture size and beamwidth.

Also, if you look on page 300 of Skolnik (Introduction to Radar Systems) about half way down the page, you will find: "Bickmore has shown that a similar result applies to a planar aperture; that is, the beamwidth in the plane of the scan varies approximately inversely as cos(theta_sub_0)". For a horizontal planar array, his theta_sub_0 is my zenith angle.

Hope this is helpful.

Jim

On 09/03/2015 06:24 PM, Dave Typinski wrote:

> Nice plots, Jim!
>
> Are you sure 1/cos is the right transform? I was thinking arccos instead.
> --
> Dave
>
> On 09/03/2015 16:10, James Abshier wrote:
>
>> Hi Dave,
>>
>> I just couldn't let go of this.
>>
>> I generated a Matlab(octave) script for plotting the squinted beam patterns from a planar array that has a circular beam pattern at broadside. I generated the circular beam contour and then stretched it out in the vertical direction as a function of 1/cos(elevation). The stretched (elliptical) contour (in the form of a set of unit vectors) was (were) transformed from antenna coordinates to East, North, Up coordinates and the azimuth and elevation of each beam contour vector were computed and plotted. Attached are two example plots. Note that azimuth in these plots increases counter-clockwise. In one plot, the azimuth was different for each squint (elevation) so that the patterns didn't overlap.
>>
>> The model is not exact, but I am guessing that it is probably not too bad except at low grazing angles where the end-fire properties of the array apply.
>>
>> Jim
>>
>> On 09/02/2015 10:13 PM, Dave Typinski wrote:
>>
>>> Hi Jim,

Did a little more research and read the Von Aulock paper referenced in the Skolnik figure caption.

You're exactly right: the plane in Skolnik is in direction cosine space, while the RJP sky view is a linear-scaled polar AZ/EL plot.

The neat thing is that /in direction cosine space/, the shape of a phased array's beam does /not/ change as the beam is steered. It is simply translated away from the origin, retaining whatever shape it has at zenith.

The real-world beam shape change with steering becomes evident when a coordinate transform is made from direction cosine space to either the unit sphere or to linear-scaled polar coordinates.

All of which means that it's simply a matter of finding the right coordinate transform equations to go from 2D direction cosine space to a linear-scaled polar AZ/EL plot, which I'm sure is quite easy.

Thanks for the thoughts, Jim!
--
Dave

On 09/01/2015 21:46, James Abshier wrote:

> Dave,
>
> Here are my thoughts on the problem.
>
> The diagram shown appears to be somewhat notional since the projection on the
> unit hemisphere would actually be the Fourier transform of the array illumination pattern. Assuming that you are talking about a phased array (as in Skolnik), then the projection of the aperture on a plane normal to boresight would be foreshortened at low boresight elevation angles (high zenith angles). This results in a broadening of the beam in elevation (due to the Fourier transform property) as shown in the figure. Since the Radio Jupiter polar plot has a linear relationship between radial distance and zenith angle, the beam pattern would be elongated in elevation approximately as a function of 1/cos(zenith angle). What it would look like exactly would be somewhat complicated to determine. I don't think that you can call the Radio Jupiter plot a projection in the same sense as the diagram shown in Skolnik. It is just an azimuth-elevation plot. Perhaps you could plot the beam pattern as a rectangle in azimuth and (elongated) elevation. It would then look like a wedge shape in the Radio Jupiter plot.
>
> Jim Abshier

On 09/01/2015 06:55 PM, Dave Typinski wrote:

> I wonder if anyone here can point me in the right direction for a geometric projection problem.
>
> Here's the deal.
>
> *Fact 1:*
>
> When you scan the beam of a phased array radar (or radio telescope) across the
> sky, the beam changes shape the further you go from the direction normal to
> the plane of the phased array.
>
> E.g., if the antenna array is planar and horizontal, and if it produces a
> circular beam when pointed at zenith, then any beam steering progressively
> changes the beam shape -- and, the closer the beam gets to the horizon, the
> worse the deformation.
>
> This is shown very well in the figure below, which is from Skolnik's excellent
> book, "Introduction to Radar Systems".
>
> *Fact 2:*
>
> Radio Jupiter Pro shows the position of objects in the sky on a polar plot. An example is shown below:
>
> Jupiter and the Sun are shown off to the east, real close together. The concentric rings are 30 degs of elevation, with zenith at the center of the image.
>
> *The problem: **
> *
> How can one correctly plot that teardrop-shaped beam (shown in the radar
> illustration) onto the polar coordinate sky map?
>
> At first, it may seem like this is simple, that when the beam is projected onto a plane, it's a circle. But this is not so.
>
> It would only be so if the polar sky map did not have even divisions in the radial direction. If the rings of elevation were spaced according to a cosine

curve, then the beam shape would be a circle. But, the spacing is linear. So
it's not a circle in polar coordinates.

But if not a circle, then what shape /is/ it in polar coordinates?

How does one translate the plane shown in the radar diagram (showing three
circles) to the plane shown in the sky map?

Any ideas?
--
Dave

--
--