

Astronomy 518

Astrometry Lecture

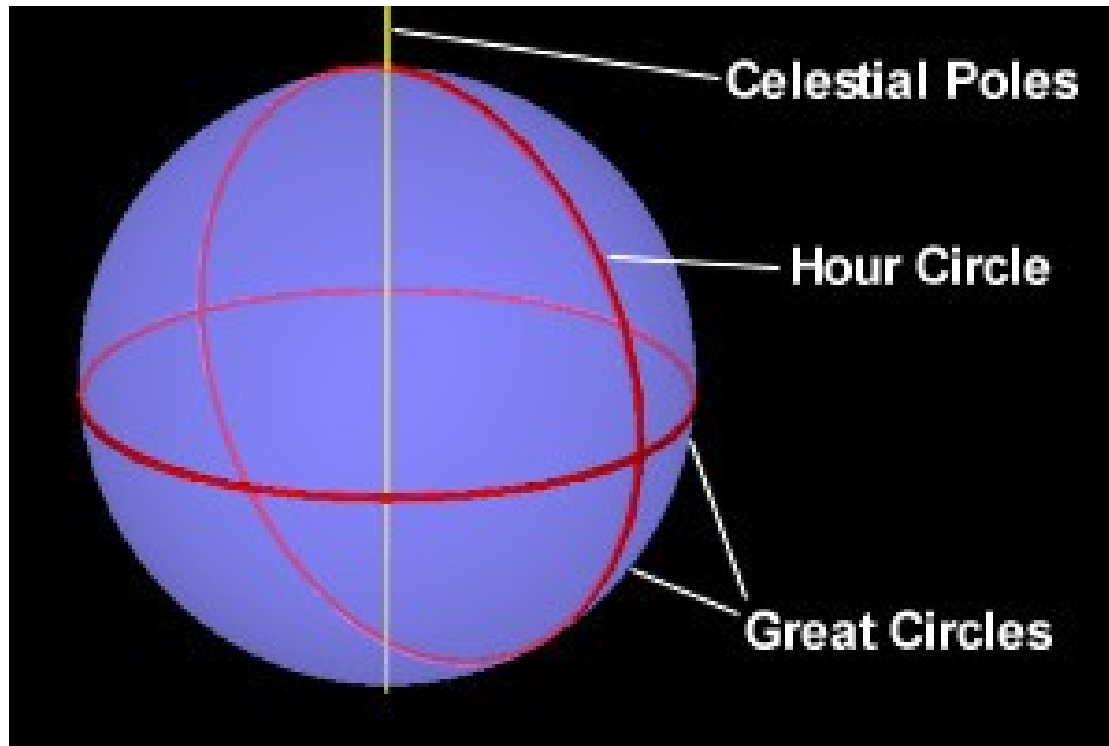
Astrometry: the branch of astronomy concerned with the measurement of the positions of celestial bodies on the celestial sphere, conditions such as precession, nutation, and proper motion that cause the positions to change with time, and corrections to the positions due to distortions in the optics, atmosphere refraction, and aberration caused by the Earth's motion.

Coordinate Systems

- There are different kinds of coordinate systems used in astronomy. The common ones use a coordinate grid projected onto the celestial sphere. These coordinate systems are characterized by a **fundamental circle**, a **secondary great circle**, a **zero point** on the secondary circle, and one of the poles of this circle.
- Common Coordinate Systems Used in Astronomy
 - Horizon
 - Equatorial
 - Ecliptic
 - Galactic

The Celestial Sphere

The celestial sphere contains any number of large circles called great circles. A great circle is the intersection on the surface of a sphere of any plane passing through the center of the sphere. Any great circle intersecting the celestial poles is called an hour circle.

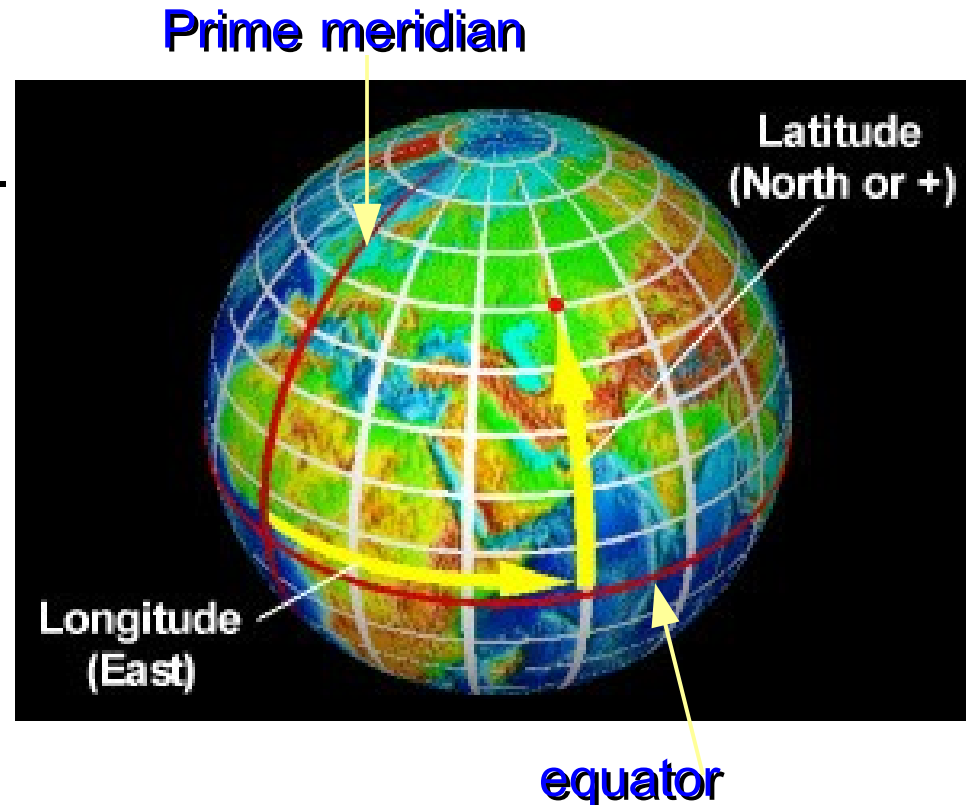


Latitude and Longitude

- The fundamental plane is the Earth's equator
- Meridians (longitude lines) are great circles which connect the north pole to the south pole.
- The zero point for these lines is the prime meridian which runs through Greenwich, England.

Latitude: is a point's angular distance above or below the equator. It ranges from 90° north (positive) to 90° south (negative).

• **Longitude** is a point's angular position east or west of the prime meridian in units ranging from 0 at the prime meridian to 0° to 180° east (+) or west (-).



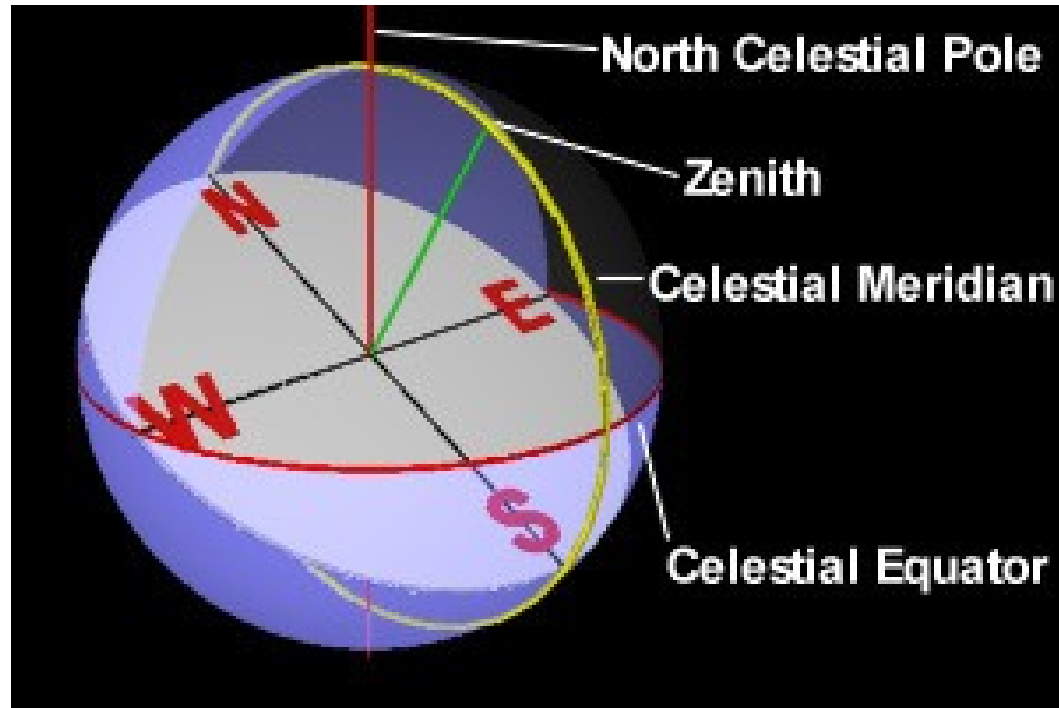
Horizon Coordinate system

Zenith: The point on the celestial sphere that lies vertically above an observer and is 90° from all points on the horizon

Nadir: The point on the celestial sphere that lies directly beneath an observer. It is diametrically opposite the zenith. The line connecting the zenith to the nadir is called the zenith-nadir axis.

The celestial meridian is great circle which intersects the zenith, the nadir, and the celestial poles.

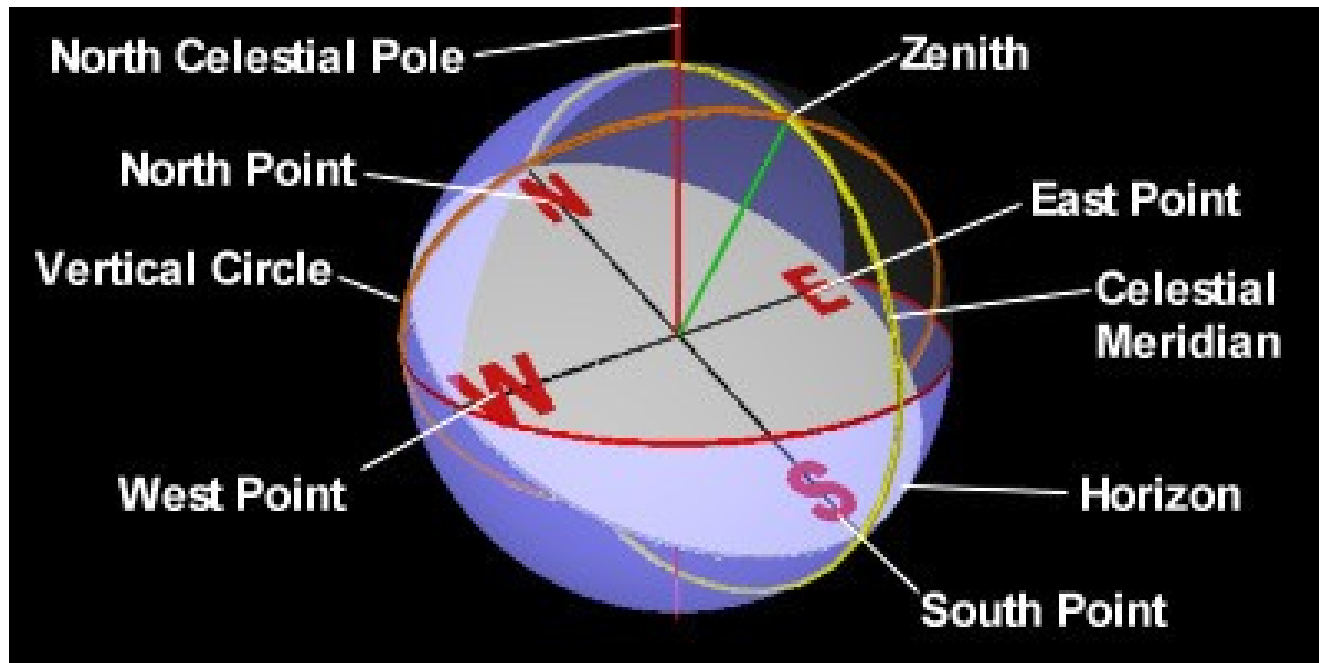
The astronomical horizon is a great circle on the celestial sphere which is perpendicular to the zenith-nadir axis.



Horizon System cont.

Cardinal points:

- North Point: intersection of the celestial meridian and the horizon that is also closest to the north celestial pole. The south point is the other intersection point
- East and West points lie at the intersections of the horizon and the celestial equator. Clockwise, the east point is 90° from the north point.



Horizon Coordinate System (alt-az system)

Fundamental circle: astronomical horizon

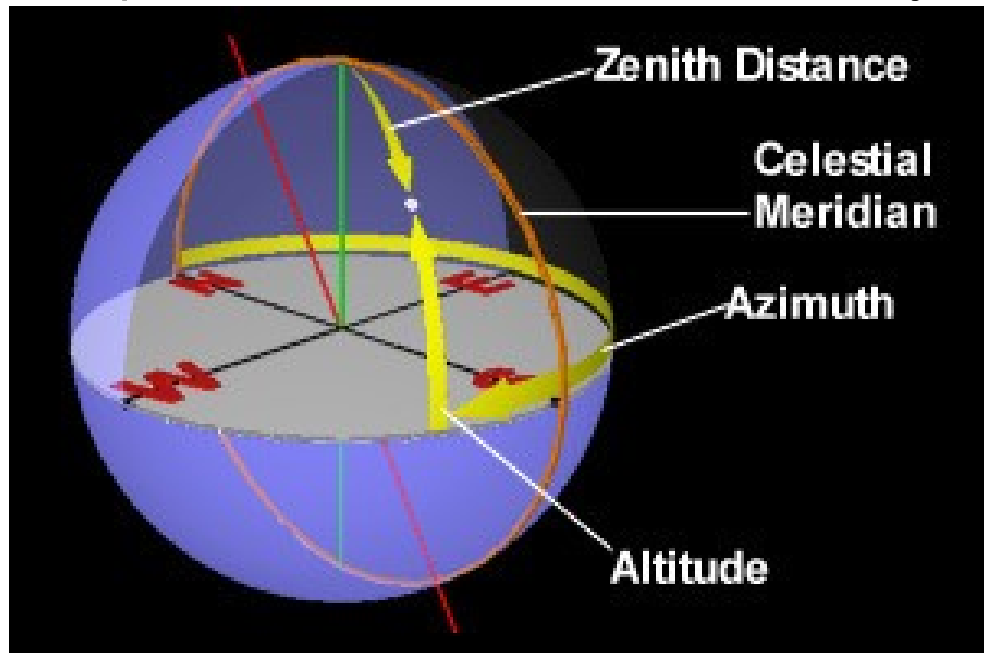
Zero point: north cardinal point

Altitude (a): is the angular distance north (+) or south (-) of the horizon. It is measured along a vertical circle through the celestial body. (0°:rising, 90°: zenith)

Vertical circle: great circles which are perpendicular to the horizon and intersect the zenith

Azimuth (A) : of a body is its angular distance measured eastwards along the horizon from the north point to the intersection of the object's vertical circle. (0° to 360°)

Zenith Distance is the complement of the altitude ($90 - a$)



Horizon Coordinate System

- Advantage: Easy to use system. It is often useful to know how high a star is above the horizon and in what direction it can be found.

Many telescopes use the alt-az mounts because of lower cost and greater stability. This means that computer control systems which can transform alt-az coordinates to equatorial coordinates are required.

- Disadvantage: because any coordinates given in the horizontal or alt-az system depends

- * *place* of observation

- (because the sky appears different from different points on Earth)

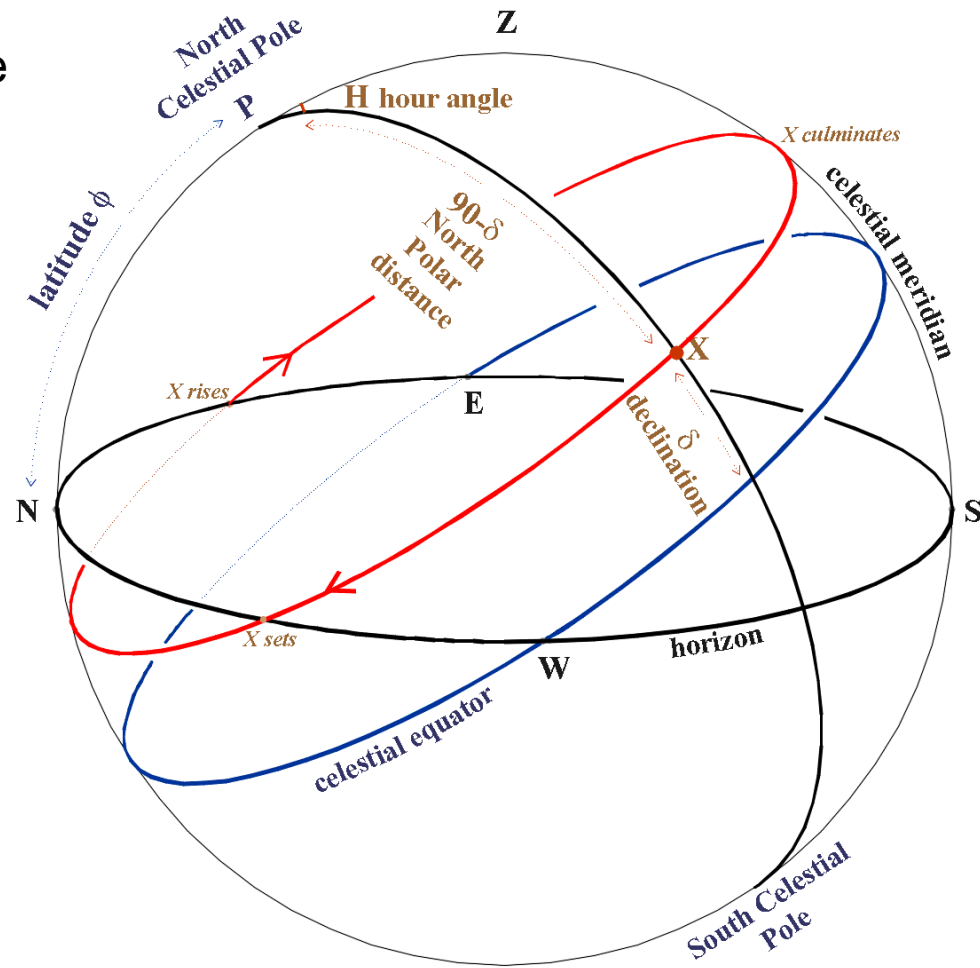
- * on the *time* of observation

- (because the Earth rotates, the zenith is always moving relative to the stars)

- We need a system of celestial coordinates which is fixed on the sky, independent of the observer's time and place. For this, we change the *fundamental circle* from the horizon to the *celestial equator*.

1st Equatorial System (Hour angle, Declination)

- The **celestial meridian** (observer's meridian) is great circle which intersects the zenith, the nadir, and the celestial poles.
- The **declination** (δ) of X is the angular distance from the celestial equator to X, measured from -90° at the SCP to $+90^\circ$ at the NCP. Any point on celestial equator has declination 0° .
- The **Hour Angle** or HA (H) of object X is the angular distance between the meridian of X and celestial meridian. It is measured westwards in hours, 0h-24h.
- This system is still dependent on the time of observation, but an object's declination generally doesn't change rapidly, and its Hour Angle can be determined quite simply, given the time and the location.



Transforming between coordinate systems

Two general methods to do this:

1. Spherical trigonometry (*Textbook on Spherical Astronomy* by W.M. Smart)
2. Using rotation matrices
 - a. Generate a vector in the original coordinate system
 - b. Convert the vector to another coordinate system by rotating the coordinates using matrix multiplication
 - c. Convert the vector to the angles of the new coordinate system

Relationship between Spherical Coordinates and Cartesian Coordinates

Spherical coordinates (spherical polar coordinates) are a system of coordinates that are used to describe positions on a sphere

θ is the angle from the x-axis

ϕ is the angle from the z-axis (called the colatitude or zenith and measured from 0 to 180°)

r is the distance from the origin,

The spherical coordinates are converted to Cartesian coordinates by

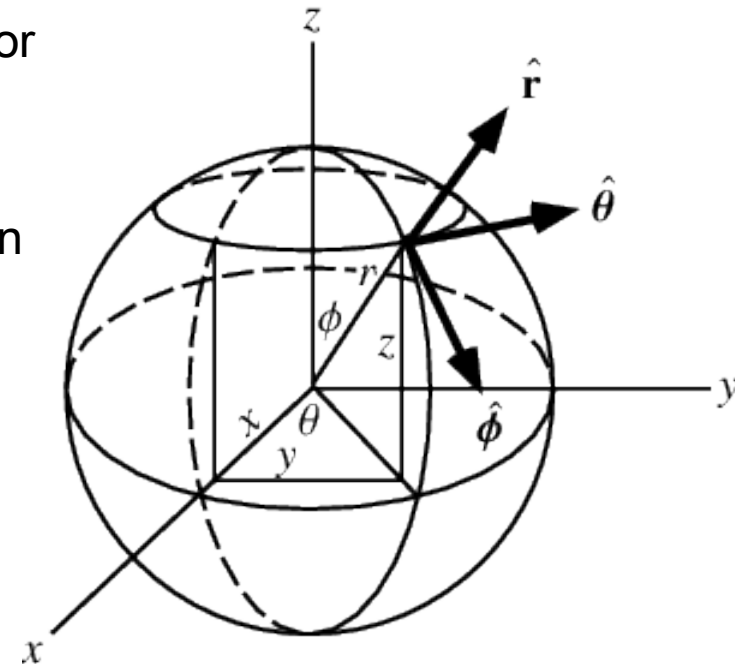
$$x = r \sin \phi \cos \theta$$

$$y = r \sin \phi \sin \theta$$

$$z = r \cos \phi$$

The position vector is $\mathbf{r} \equiv \begin{bmatrix} r \cos \theta \sin \phi \\ r \sin \theta \sin \phi \\ r \cos \phi \end{bmatrix}$

The unit position vector $\hat{\mathbf{r}} = \begin{bmatrix} \cos \theta \sin \phi \\ \sin \theta \sin \phi \\ \cos \phi \end{bmatrix}$

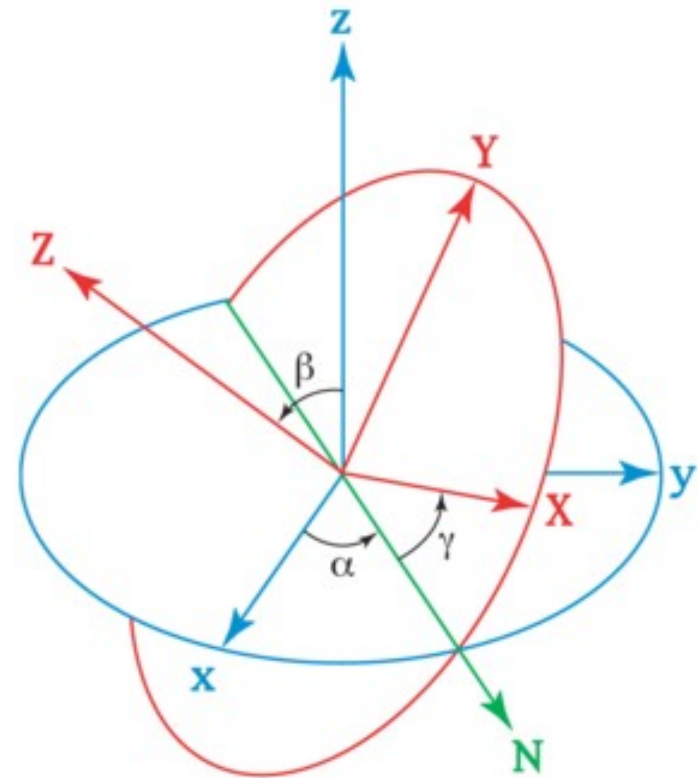


Position vector in h system = $\begin{bmatrix} \cos a \sin z \\ \sin a \sin z \\ \cos z \end{bmatrix}$

Transforming Between Coordinate Systems

It is also useful to have an understanding of *Euler rotations* and *Euler Angles*, which are a way to determine the transformation from one coordinate system to another defined on the celestial sphere.

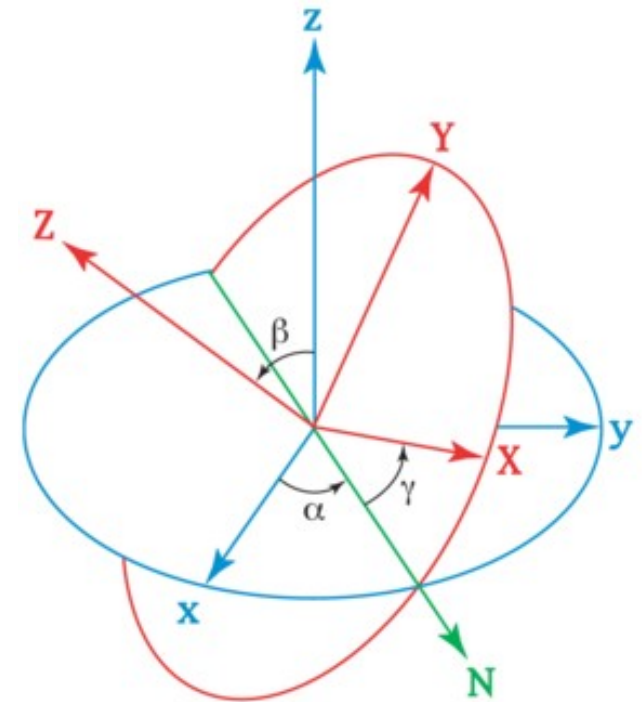
- The intersection of the xy and the XY coordinate planes is called the *line of nodes* (N).
- α is the angle between the x -axis and the line of nodes.
- β is the angle between the z -axis and the Z -axis.
- γ is the angle between the line of nodes and the X -axis.
- Angle ranges:
 - α and γ range from 0 to 360 degrees
 - β ranges from 0 to 180 degrees



Euler angles - The xyz (fixed) system is shown in blue, the XYZ (rotated) system is shown in red. The line of nodes, labelled N, is shown in green.

Fixed axes of rotation - The xyz system is fixed while the XYZ system rotates. Start with the rotating XYZ system coinciding with the fixed xyz system.

- Rotate the XYZ -system about the Z -axis by γ . The X -axis is now at angle γ with respect to the x -axis.
- Rotate the XYZ -system again about the X -axis by β . The Z -axis is now at angle β with respect to the z -axis.
- Rotate the XYZ -system a third time about the Z -axis by α .
- (Note that the first and third axes are identical). These three angles α , β , γ are the *Euler angles*



Euler angles - The xyz (fixed) system is shown in blue, the XYZ (rotated) system is shown in red. The line of nodes, labelled N , is shown in green.

Expressed in rotation matrices this transformation is represented by

$$[\mathbf{R}] = \begin{bmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation Matrices

The three basic rotations about the x,y, or z axes by angles α , β , and γ are equivalent to multiplication by the matrices:

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transformation between Horizon system to Hour angle and declination

The transformation from hour angle (h) and declination (δ) to horizon system: azimuth (a), zenith distance (z)

$$I(a,z) = \mathbf{R}_3(180^\circ) \mathbf{R}_2(90^\circ - \varphi) I(h,\delta),$$

where φ is the observer's geographic latitude

$$\sin z \cos a = \cos \varphi \sin \delta - \sin \varphi \cos \delta \cos h$$

$$\sin z \sin a = -\cos \delta \sin h$$

$$\cos z = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos h$$

The inverse relationship is:

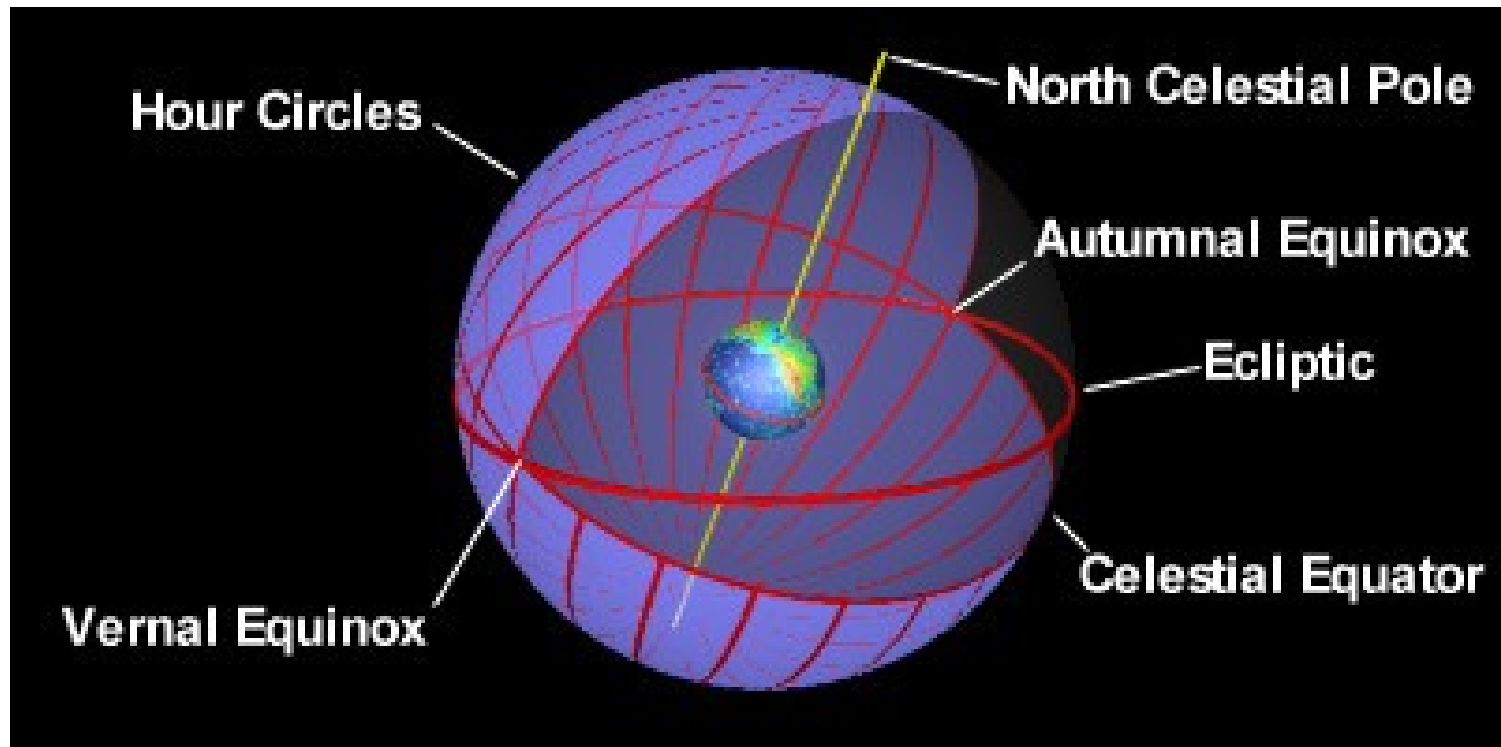
$$\cos \delta \cos h = \cos \varphi \cos z - \sin \varphi \sin z \cos a$$

$$\cos \delta \sin h = -\sin z \sin a$$

$$\sin \delta = \sin \varphi \cos z + \cos \varphi \sin z \cos a$$

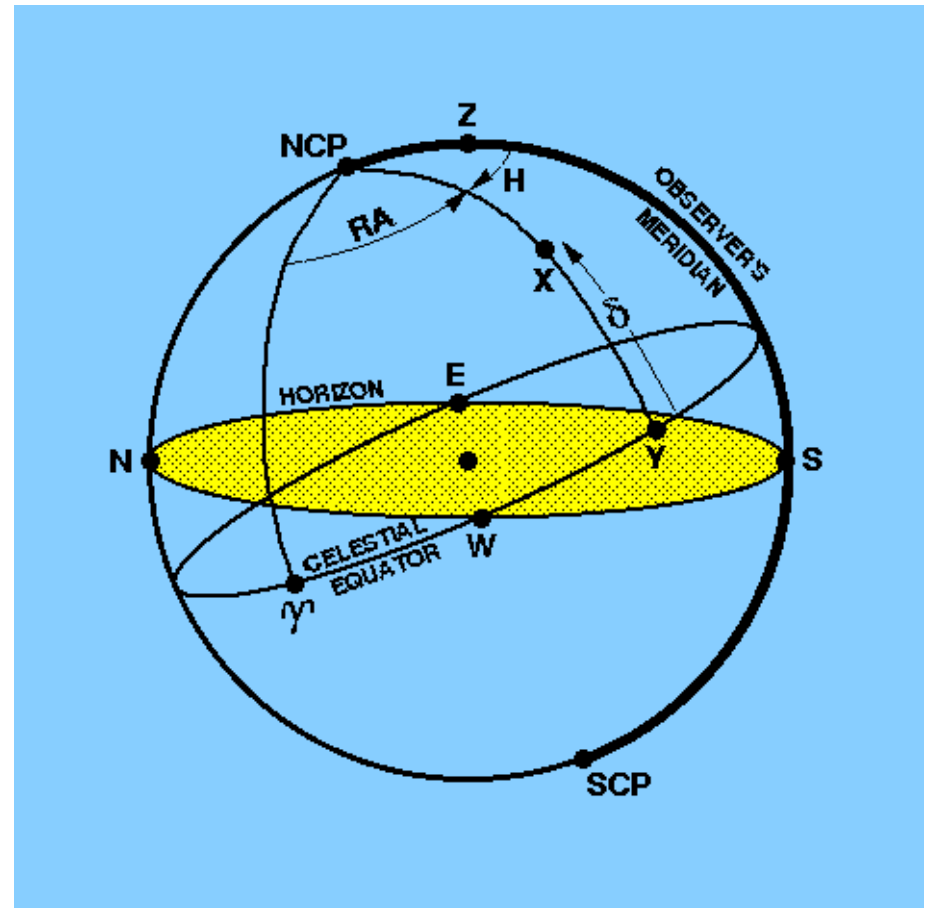
Celestial sphere, ecliptic and vernal equinox

- The **ecliptic** is the apparent path of the Sun in the sky. The obliquity of the ecliptic is $23^{\circ} 26'$.
- Intersection of the ecliptic and celestial equator define the **vernal equinox** and **autumnal equinox**
- **Hour circle**: a great circle that passes through celestial poles and is perpendicular to the celestial equator.



2nd Equatorial System (Right Ascension and Declination)

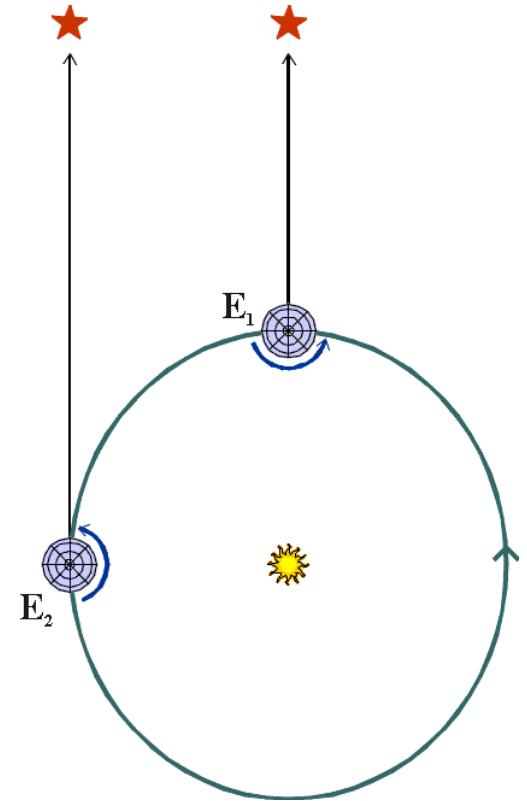
- Most widely used coordinate system (for objects outside the solar system).
- Fundamental reference circle: celestial equator
- Zero point: vernal equinox.
- The right ascension of a point is the angular distance measured eastward along the celestial equator between vernal equinox and the hour circle intersecting the point. It is measured in hours, minutes, and seconds from 0 to 24 hours.
- The declination of a point is the angular distance above or below the celestial equator. It is measured in degrees, minutes, and seconds from -90° to $+90^\circ$ with 0° being on the celestial equator. Negative degrees indicate that the point is south of the celestial equator, while positive degrees indicate that it is north of the celestial equator.



Sidereal Time

Which stars are on your local meridian?

- It depends on the time at which you observe. In fact, it depends on both the *date* and the (clock) time, because the Earth is in orbit around the Sun.
- Consider the Earth at position E_1 on the diagram. The star shown is on the meridian at midnight by the clock. But three months later, when the Earth reaches position E_2 , the same star is on the meridian at 6 pm. by the clock.
- Our clocks are set to run (approximately) on **solar time** (sun time). But for astronomical observations, we need to use **sidereal time** (star time).



Consider the rotation of the Earth relative to the stars.

We define one rotation of Earth as one sidereal day, measured as the time between two successive meridian passages of the same star.

Because of the Earth's orbital motion, this is a little shorter than a solar day.

(In one year, the Earth rotates 365 times relative to the Sun, but 366 times relative to the stars. So the sidereal day is about 4 minutes shorter than the solar day.)

Hour Angle and Time

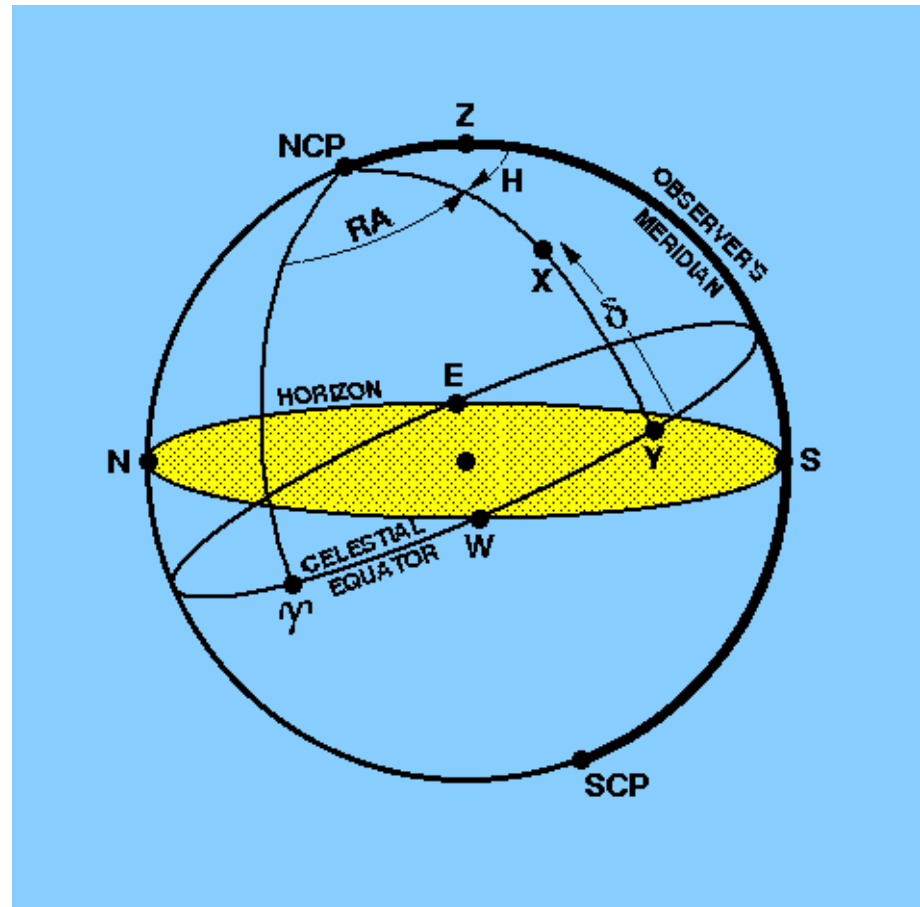
Hour angle : is the angle measured westwards along the celestial equator from the observer's meridian to the hour circle of the celestial body.

Another way to think about it is: the *hour angle* of a celestial body: is the time elapsed since the celestial body's last transit of the observer's meridian.

If $HA = -1$ (it will be 1 hour before the object is on the meridian. This is the same as $HA = 23$).

If $HA = 0$ (it is on the meridian)

If $HA = 2$ (it has been 2 hours since the object was on the meridian.



Local Sidereal Time and Right Ascension

The **Local Sidereal Time (LST) = Hour Angle of the vernal equinox.**

LST is 0 hours when the vernal equinox is on the observer's local meridian.

- Lets suppose that $LST = 1h$.
This means that the vernal equinox has moved 15° (1h) west of the meridian, and now some other star X is on the meridian.
But the Right Ascension of star X is the angular distance from the vernal equinox to X = 1h = LST.
So at any instant, **Local Sidereal Time = Right Ascension of whichever stars are on the meridian.**
- And in general, **the Hour Angle of a star = Local Sidereal Time - RA of the star.**

Transforming between Horizon to Equatorial Coordinates (and vice versa)

We assume we know φ and LST

To convert from equatorial to horizontal coordinates: Given RA, α , and declination, δ , we have

$$HA = LST - RA, \text{ in hours}$$

Therefore we have HA, declination and use the formulas from preceding page to work out the azimuth and zenith distance.

Now for the inverse problem:

to convert from horizontal to equatorial coordinates:

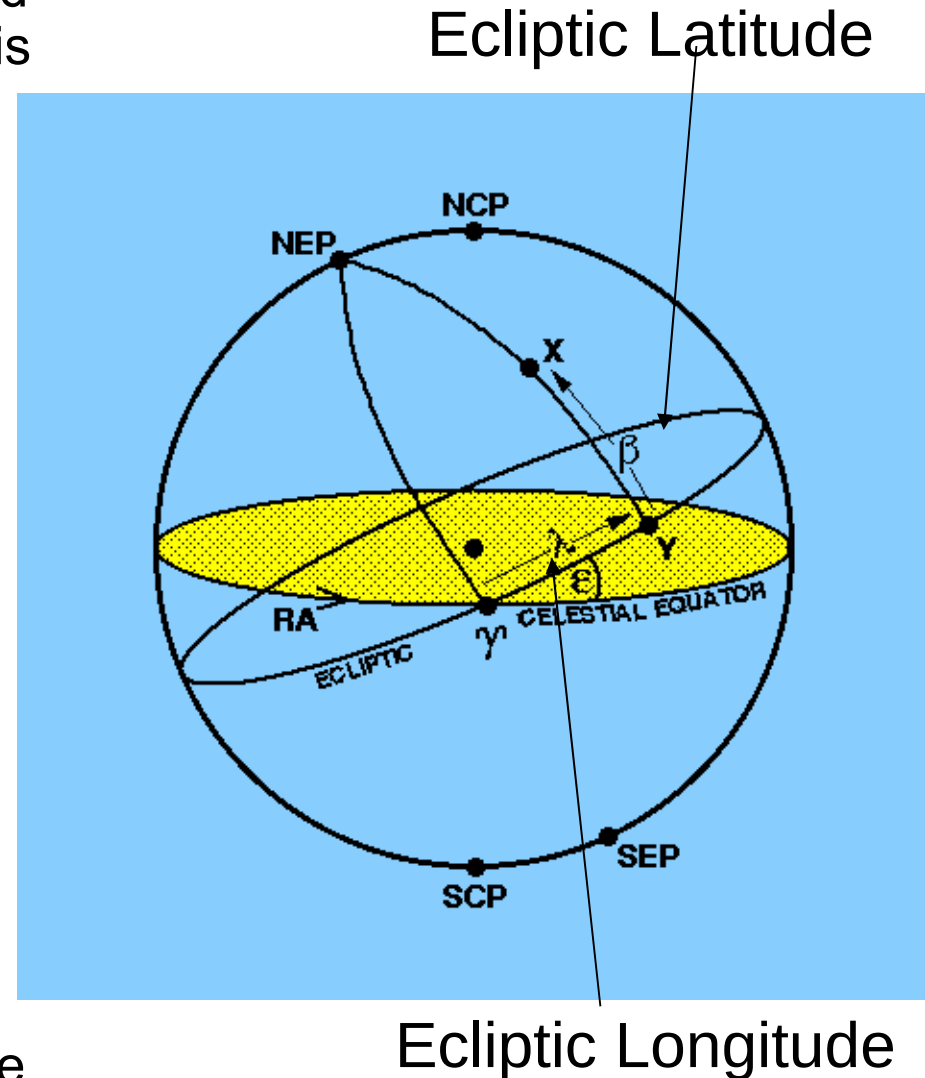
Given φ , a and A , what are α and δ

Use the formulas on the preceding page to convert, φ , A , a or ($z = 90 - a$) to δ, HA

Then find the RA using the formula $RA = LST - HA$

Ecliptic Coordinate System

- When dealing with the positions and motions of solar system objects, it is often more convenient to refer positions to the mean orbital plane of the solar system using **ecliptic coordinates**.
- Fundamental circle is the ecliptic.
- Zero point: vernal equinox.
- **Ecliptic latitude**, β , is analogous to declination, but measures distance north or south of the ecliptic, attaining $+90^\circ$ at the *north ecliptic pole (NEP)* and -90° at the *south ecliptic pole (SEP)*.
- **Ecliptic longitude**, λ , is analogous to right ascension and is measured from the vernal equinox, in the same direction as right ascension but along the ecliptic rather than the celestial equator.



Transforming Between Equatorial and Ecliptic Coordinates

The transformation from right ascension, α , and declination (δ) to ecliptic coordinates, λ , β (rotation

$$I(\lambda, \beta) = R_1(\varepsilon) I(\alpha, \delta),$$

Where $\varepsilon = 23.439\ 281^\circ$ is the obliquity of the ecliptic

$$\cos \lambda \cos \beta = \cos \alpha \cos \delta$$

$$\sin \lambda \cos \beta = \sin \varepsilon \sin \delta + \sin \alpha \cos \delta \cos \varepsilon$$

$$\sin \beta = \cos \varepsilon \sin \delta - \sin \alpha \cos \delta \sin \varepsilon$$

And the inverse relations are:

$$\cos \alpha \cos \delta = \cos \lambda \cos \beta$$

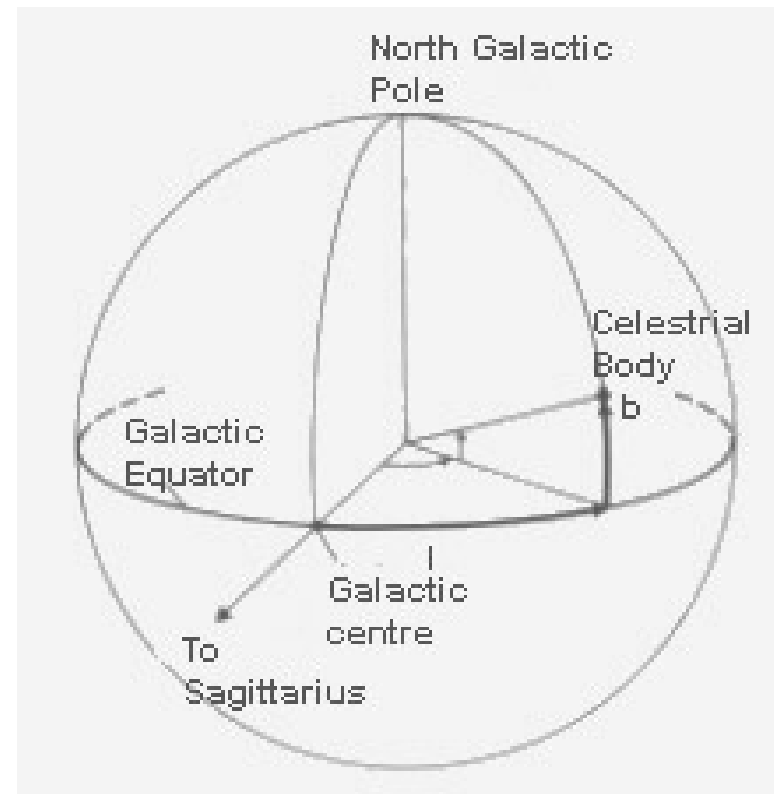
$$\sin \alpha \cos \delta = \cos \varepsilon \sin \lambda \cos \beta - \sin \varepsilon \sin \beta$$

$$\sin \delta = \sin \varepsilon \sin \lambda \cos \beta + \cos \varepsilon \sin \beta$$

Galactic Coordinate System

The equatorial system is geocentric and thus provides an inappropriate viewpoint for problems of Galactic structure and dynamics.

- The fundamental circle is galactic equator (the great circle on the celestial sphere that represents the path of the Milky Way Galaxy. The plane is inclined at an angle of 62.87 degrees to celestial equator.)
- Zero point lies in the direction of the galactic center as seen from Earth
- Galactic latitude (b) of a celestial body is its angular distance north (+) or south (-) of the galactic equator (it ranges from 0 to 90°)
- Galactic Longitude (l) of a celestial body is its angular distance (from 0 to 360°) from the nominal galactic center measured eastwards along the galactic equator to the intersection of the great circle passing through the body.



Converting between galactic to equatorial coordinates

The galactic north pole is at RA = 12:51.4, Dec = +27:07 (2000.0), the galactic center at RA = 17:45.6, Dec = -28:56 (2000.0).

The inclination of the galactic equator to the celestial equator is thus 62.9° .

The intersection, or node line, of the two equators is at RA = 282.25° , Dec = 0:00 (2000.0), and at $l = 33^\circ$, $b=0$.

The transformation between the equatorial and galactic systems consisted of:

1. a rotation around the celestial polar axis by 282.25° , so that the reference zero longitude matches the node
2. a rotation around the node by 62.9°
3. a rotation around the galactic polar axis by 33° so that the zero longitude meridian matches the galactic center.

Transforming between galactic to equatorial coordinates

$$\begin{aligned}\cos b \cos(l-33^\circ) &= \cos \delta \cos(\alpha - 282.25^\circ) \\ \cos b \sin(l-33^\circ) &= \sin \delta \sin(62.6^\circ) + \cos \delta \sin(\alpha - 282.25^\circ) \cos(62.6^\circ) \\ \sin b &= \sin \delta \cos(62.6^\circ) - \cos \delta \sin(\alpha - 282.25^\circ) \sin(62.6^\circ)\end{aligned}$$

The reverse transformations are:

$$\begin{aligned}\cos \delta \cos(\alpha - 282.25) &= \cos b \cos(l - 33^\circ) \\ \cos \delta \sin(\alpha - 282.25) &= \cos b \sin(l - 33^\circ) \cos(62.6^\circ) - \sin b \sin(62.6^\circ) \\ \sin \delta &= \cos b \sin(l - 33^\circ) \sin(62.6^\circ) + \sin b \cos(62.6^\circ)\end{aligned}$$

Note: precession causes changes in α and δ

Longitude Zero Point of Galactic Coordinate System

- The compact radio source, Sagittarius A* is now believed to mark the position of the Galactic nucleus, actually lies about 5 arc minutes away from the nominal galactic center.
- The position of the zero galactic longitude (nominal galactic center) was agreed on by the IAU in 1959 to be at RA 17 h 42.4m, dec -28.55 (epoch 1950). Before 1958 the zero point was at RA 17 h 45.6 m, Dec $-28.56.2$. The new system is designated by a superior Roman numeral II (i.e. b^{II} , l^{II}) and the old system by the superior Roman number I. The old system measure Galactic longitude from one of points where the Galactic equator intersects the celestial equator.

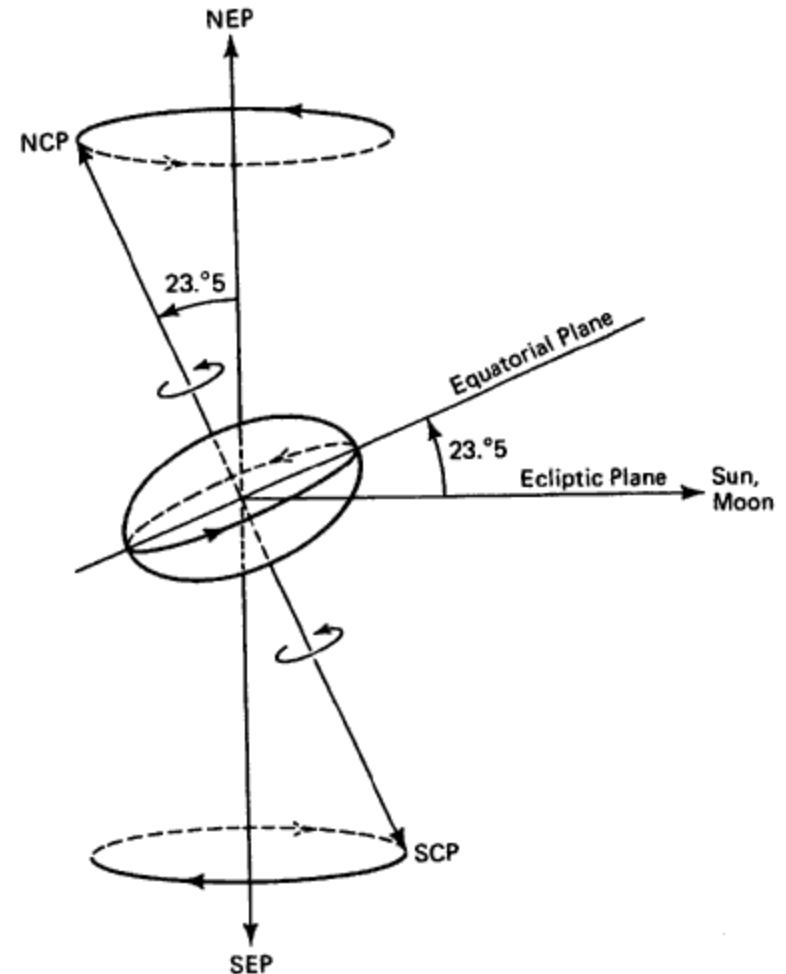
Changes of Celestial Coordinates

- There are several effects that cause the coordinates of a star to deviate from those given in star catalogs.
 - Precession
 - Nutation
 - Proper Motion
 - Parallax
 - Refraction

Earth's Axial Precession

The Earth's axis is not fixed in space, but like a spinning top, the direction of the rotation axis executes a slow precession with a period of 26,000 years.

Caused by: the Moon and the Sun exert tidal forces on the equatorial bulge of the Earth, and there are small effects from the rest of the planets in the solar system. This complex set of forces results in the North and South Celestial Poles to circling around the Ecliptic Poles.



Precession

This combined movement has short and long period components which are usually broken down into 'corrections' as.

- luni-solar precession (period about 26,000 years)
- planetary precession (shift of ecliptic due to orbits of planets)
- nutation (major period 18.6 years and amplitude 9.5 arc-seconds)
- polar motion (due to internal dynamics of Earth, and very small)

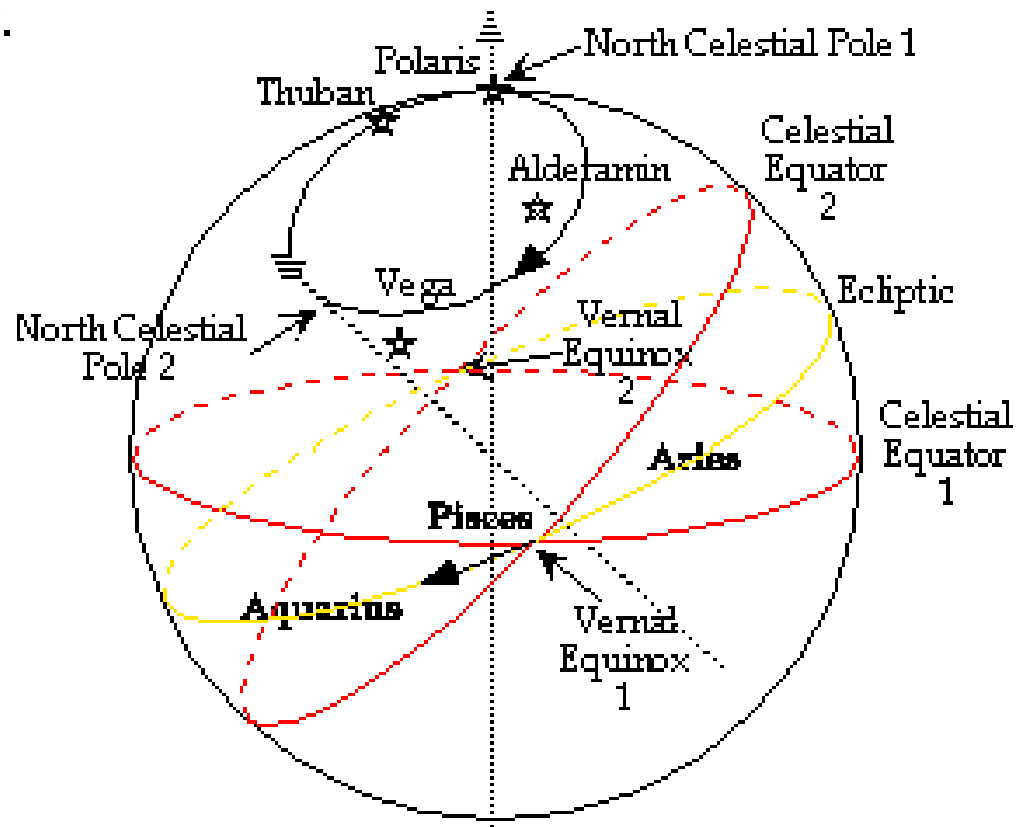
The two precession effects are lumped together into the *general precession*. The annual general precession is given by $50.2564 + 0.0222T$ arc seconds, where T is the time in Julian centuries from 1900.00

Planetary precession also acts to change the obliquity of the ecliptic of the Earth (between 21.5 and 24.5 degrees) over a 41,000 year cycle. Currently the obliquity of the ecliptic is being reduced by 0.5 arc seconds/year.

Precession of Equinoxes

As the precession of the axis occurs, the orientation of the celestial equator will also change, since it is perpendicular to the axis. However, the ecliptic is fixed on the sphere. As a result, the intersections between the celestial equator and the ecliptic (the equinoxes) move along the ecliptic at a rate of about 50.27 arc seconds per year.

Two different positions, now (1) and 13,000 years in the future (2).



Effects of Precession of Equinoxes

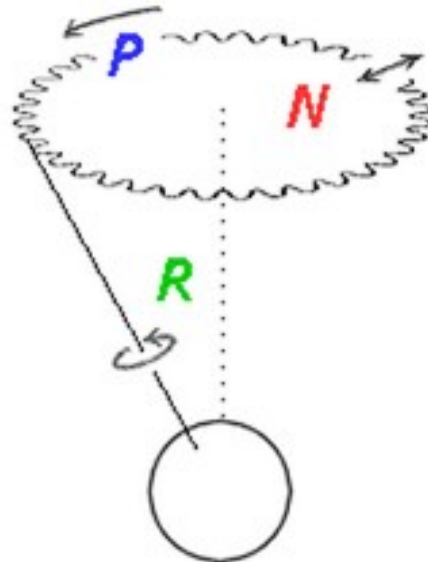
- If the position of the celestial poles and equators are changing on the celestial sphere, then the celestial coordinates (α, δ) of objects, which are defined by the reference of the celestial equator and celestial poles, are also constantly changing.
- The effects are very noticeable (50.27) arc secs a year along the ecliptic.
- Since the location of the equinox changes with time, coordinate systems that are defined by the vernal equinox, must have a date associated with them. This specified year is called the **Equinox** (not epoch).
- The common practice is to use standard years (like 1950, 2000, 2050) to cite the Equinox. Currently we use Equinox J2000.0

Three main epochs are in common use;

- B1950.0 - the equinox and mean equator of 1949 Dec 31st 22:09 UT. (B refers to Besselian year, years are based on a solar year)
- J2000.0 - the equinox and mean equator of 2000 Jan 1st 12:00 UT (J refers to Julian year which are 365.25 days)
- Equinox of the date means that the equinox is the same as the epoch (allowing for nutation).

Nutation

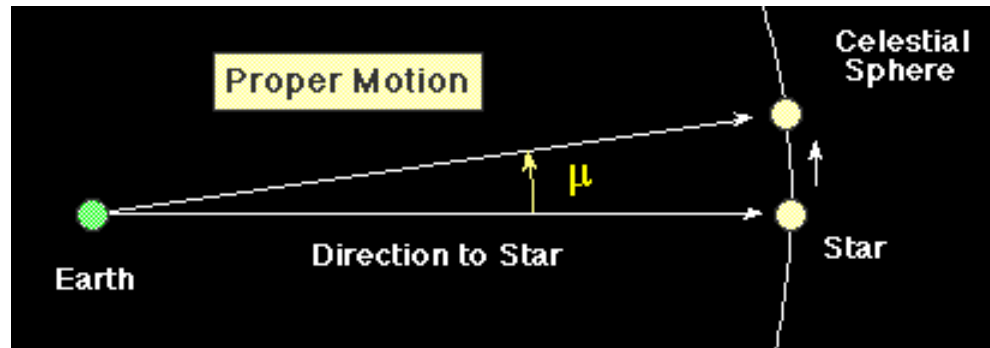
- **Nutation** is a small cyclical motion superimposed upon the 26,000-year precession of the Earth's axis of rotation. It is mainly caused by the gravitational attraction of the Sun and Moon, which continuously change location relative to each other and thus cause nutation in Earth's axis. The largest component of Earth's nutation has a period of 18.6 years, the same as that of the precession of the Moon's orbital nodes.
- Because the dynamics of the planets are so well known, nutation can be calculated within seconds of arc over periods of many decades.



Proper Motion

Stars are not fixed in space but move according to their space velocity and the gravitational field in their environment.

Proper Motion is the apparent motion of a star across the celestial sphere at right angles to the observer's line of sight; any radial motion (toward or away from the Sun) is not included. It is observed with respect to a framework of very distant background stars or galaxies.



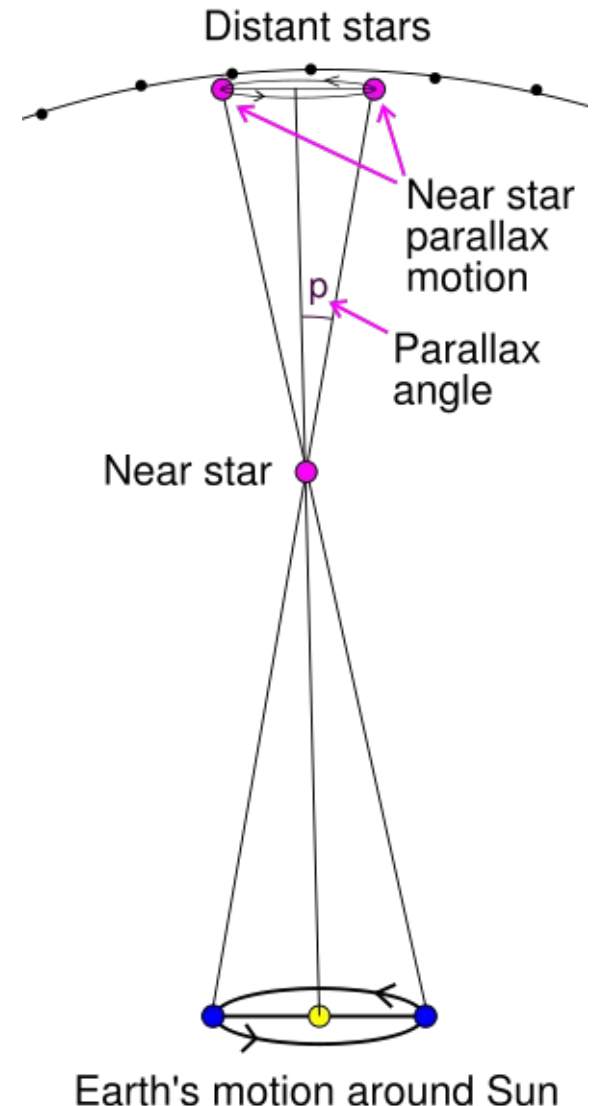
Proper motion is generally measured in seconds of arc per year; the largest known is that of Barnard's Star 10.27 "/y (arc seconds per year). Only about 500 stars are known to have proper motions of more than 1 "/y

The radial component (which changes the distance) can be measured with much higher accuracy in the Doppler shift of spectral lines visible in the spectra of stars

Parallax

- Parallax is the angular displacement in the apparent position of a celestial body when observed from two widely separated points.
- The annual parallax is defined as the difference in position of a star as seen from the Earth and Sun.
- A parsec is the distance for which the annual parallax is 1 arc-seconds. A parsec equals 3.26 light years.
- The distance of an object (in parsecs) can be computed as the reciprocal of the parallax:

$$p = 1/d \text{ arc seconds, when the distance is given in parsecs}$$



Atmospheric Refraction

- Atmospheric refraction is the deviation of light (or other electromagnetic radiation) from a straight line as it passes through the atmosphere due to the variation in air density as a function of altitude.
- Atmospheric refraction causes astronomical objects to appear higher in the sky than they are in reality. This effect even lifts objects from up to 35 arc minutes below horizon to above the apparent horizon.



The index of refraction, n , is the ratio of the speed of light in a vacuum, c , to the speed of light in the medium, v

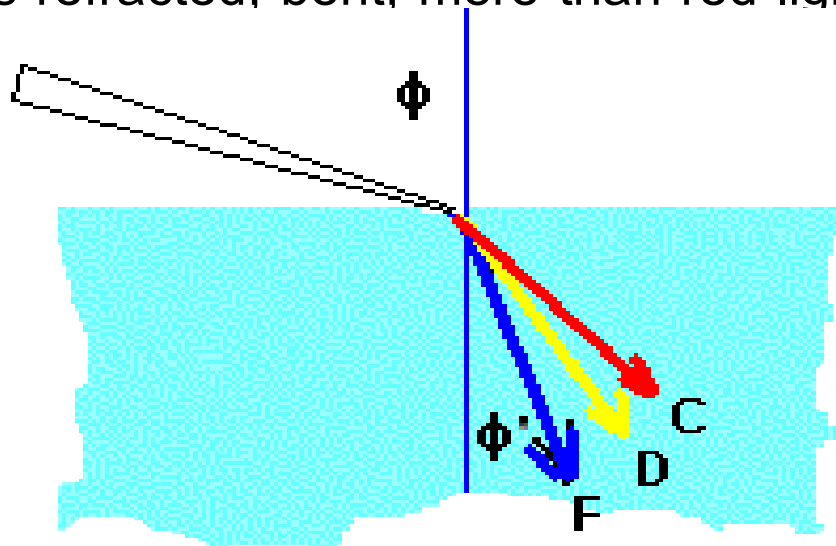
$$n = c/v$$

The more dense the material the slower the speed of light in that material, Thus $n > 1$ for all materials.

Atmospheric refraction and dispersion

The frequency of light, f , does not change when it passes from one medium to another. So according to the formula, $v = \lambda f$, the wavelength must change. The index of refraction can therefore be written in terms of wavelength as $n = \lambda_{\text{vacuum}} / \lambda_{\text{medium}}$.

This causes light to be refracted by different amounts according to the wavelength. The shorter the wavelength the greater the index of refraction (blue light is refracted, bent, more than red light)



Reference Systems & Frames

The concept of position is a relative one. One can define a position only with respect to something. In astronomy that is a **reference system**, which is a theoretical concept, or a **reference frame**, a practical realization of a reference system.

- A **reference system** is the complete specification of how a celestial coordinate system is to be formed. It defines the origin, fundamental planes, it specifies all of the constants, models and algorithms used to transform between observable quantities and the reference data that conform to the system.
- A **reference frame** is the practical realization of the reference system, using as a catalog of positions and proper motions (if measurable) of stars or extragalactic objects.

The fundamental plane has conventionally been the celestial equator and vernal equinox is the zero point for right ascension. The theories of the motion of the Earth that define how the ecliptic and equatorial planes move are imperfect. The exact location of the vernal equinox is difficult to determine.

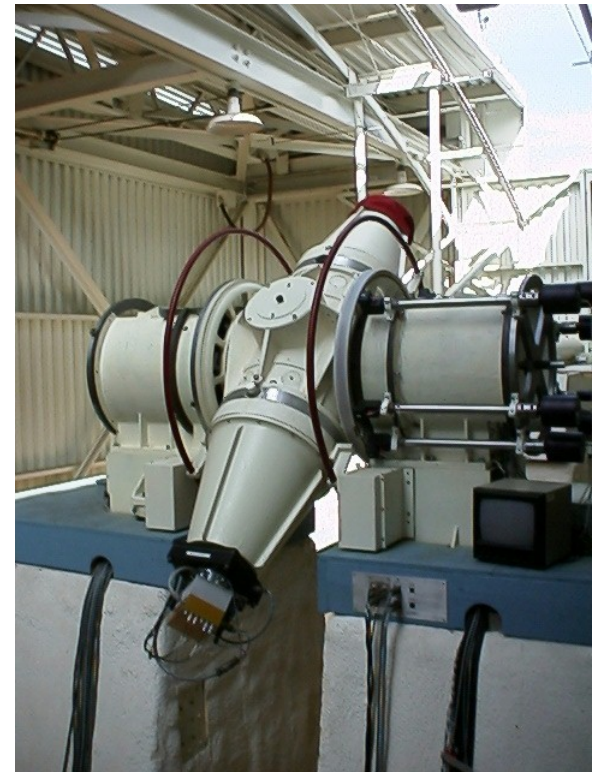
Meridian circle telescope & reference frames

The meridian circle, or transit circle, is a specialized telescope that can only be moved along the meridian (North-South arc passing through the zenith). Observations are made by timing when a star passing the meridian, at the same time measuring its angular distances from the zenith.

These telescopes rely on the rotation of the Earth to bring objects into their field of view.

Knowing the geographic latitude and longitude these measurements can be used to derive the star's right ascension and declination.

Until 1997 reference frames were based on observations taken with meridian circle telescopes.



USNO Flagstaff Astrometric Scanning Transit Telescope (FASTT)

Reference Frames

The establishment of celestial reference frames is coordinated by the International Astronomical Union (IAU).

Previous astronomical reference system: Fifth Fundamental Catalog (FK5), J2000 (1988) (FK4 B1950, 1963)

- A catalog of 1535 bright stars (to magnitude 7.5), supplemented by a fainter extension of 3117 additional stars (to magnitude 9.5)
- Compiled from meridian observations taken in the visual band
- The B1950 and J2000 reference frames are defined by the mean orientation of the Earth's equator and ecliptic at the beginning of the years 1950 and 2000. The assumed orientation of the Earth on these two dates is more a matter of definition than actual since the mean orientation does not include short term motions of the earth's spin axis (nutation and other smaller effects)
- The catalogs list mostly nearby stars so any definition of coordinates tied to these catalogs is subject to errors due to motions of stars on the sky.

B1950 to J2000

- The difference between epoch and equinox is that the equinox addresses changes in the coordinate system, while the epoch addresses changes in the position of the celestial body itself. The equinox defines (partly) which coordinate system is used. The epoch defines (completely) which moment observations or predictions are valid
- Converting from B1950 to J2000.0

Many astronomy software packages have B1950 -> J2000 routines (and vice versa).

- Apply corrections for differences between FK4-FK5 (systematic error, Besselian to Julian Year, and other differences in systems)
- Precession
- Proper motion

VLBI observations and a new reference frame

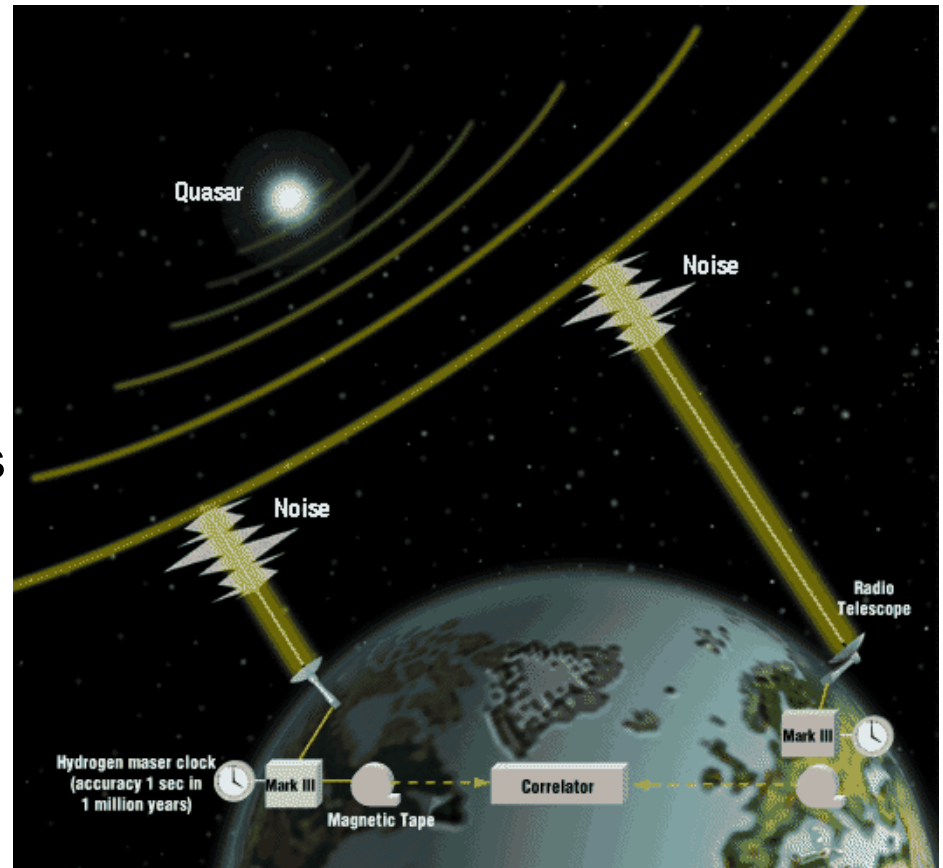
In recent years, the most precise wide-angle astrometry has been conducted with radio observations, using the techniques of Very Long Baseline Interferometry (VLBI).

- Uncertainties in radio source positions listed in all-sky VLBI catalogs are typically less than 1 milliarcsecond (and often a factor to 10 better)
- These radio sources are very distant extragalactic objects (mostly quasars) that are not expected to show measurable intrinsic motion.
- VLBI observations revealed that models of the Earth's precession and nutation (that were part of the FK5 system) were inadequate for modern astrometric precision. (The “constant of precession” has been over-estimated by about 0.3 arc seconds per century)
- Success of Hipparcos astrometric satellite (launch 1989) lead to a new set of very accurate star coordinates in the optical regime.
- In 1988 the IAU began working on a new fundamental reference.

VLBI is a geometric technique

VLBI measures the time difference between the arrival at two Earth-based antennas of a radio wavefront emitted by a distant quasar. Using large numbers of time difference measurements from many quasars observed with a global network of antennas, VLBI determines the inertial reference frame defined by the quasars and simultaneously the precise positions of the antennas.

Because the time difference measurements are precise to a few picoseconds, VLBI determines the relative positions of the antennas to a few millimeters and the quasar positions to fractions of a milliarcsecond. Since the antennas are fixed to the Earth, their locations track the instantaneous orientation of the Earth in the inertial reference frame. Relative changes in the antenna locations from a series of measurements indicate tectonic plate motion, regional deformation, and local uplift or subsidence.



International Reference System (ICRS)

The ICRS is the fundamental celestial reference system adopted by the IAU in 1997.

- Axes are fixed with respect to a set extragalactic objects which are assumed to have no measurable proper motion.
 - The reference frame is based on a kinematical rather than dynamical definition by using quasars and active nuclei of galaxies for its materialization.
 - In an ideal *dynamical* celestial reference frame, the celestial bodies move in such a way that the equations of motion have no kinematic acceleration. Dynamical reference frames are defined by the dynamics of the solar system (e.g. ephemerides of the Earth-Moon-planetary system). These reference frames have time-dependent orientation.
 - An ideal *kinematic* celestial reference frame assumes that there exists in the Universe a class of objects which have no global systematic motions and therefore are not rotating in the mean. Quasars and other distant extragalactic objects are in practice used as adequate fiducial points.

The axes are consistent, to better than 0.1 arc seconds, with the equator and equinox of J2000.0 defined by the dynamics of the Earth. However, the axes are meant to be regarded as fixed directions in space that have an existence independent of the dynamics of the Earth.

International Reference Frame (ICRF)

- International Celestial Reference Frame (ICRF) is based on radio positions of 608 extragalactic sources observed with the VLBI distributed over the entire sky.
 - Most have faint optical counterparts and the majority of quasars.
 - 212 of the 608 sources are defining sources that establish the orientation of the ICRS axes, with the origin at the solar system barycenter.
 - The positional uncertainty of the defining sources are on the order of 0.5 milliarcsecond.
 - The axes correspond closely to the “equator and equinox” of J2000.0
- The ICRS is realized at optical wavelengths by stars in the Hipparcos Catalog (118,218 stars, some as faint as 12). Only stars with well-determined proper motions are used for the ICRS realization.
- A new set of standard algorithms were adopted in 2000 to transform the ICRS catalog data to observable quantities (and vice versa). These include precession-nutation model, a new definition of the celestial pole, and a new reference point for measuring the rotation angle of the Earth around its instantaneous axis.

Reference Star Catalogs

Star catalog contain a list of stars according to position and magnitude and in some cases other properties (e.g. proper motion, spectral type, and parallax).

- **Hipparcos** contains positions, proper motions, parallaxes, and B and V magnitudes for 118,218 stars. Complete to $V = 7.3$ Positional accuracy, 1-3 mas at epoch 1991.25. Proper motion accuracy around 1 to 2 mas/yr.
- **TYCHO-2** contains position, proper motions and two-color photometric data for 2,539,913 stars. The catalog is 99% complete to $V = 11.0$. Positional accuracies 10 to 100 mas and proper motions accuracies from 1 to 3 mas.
- **USNO B 1.0** a catalog of over 1 billion objects. The Tycho-2 catalog is the astrometric reference. All-sky coverage down to $V = 21$, 200 mas accuracy at J2000, 0.3 magnitude photometric accuracy in up to five colors, and 85% accuracy for distinguishing stars from non-stellar objects.
- **Sloan Digital Sky Survey (SDSS)** – mapped one-quarter of the sky in 5 optical bandpasses, and obtained spectra of galaxies and quasars. The 6th data release contains 287 million objects.
- **2MASS**– Two Micron All Sky Survey. Point source catalog contain 470 million objects, mostly stars. The extended source catalog contains data on 1.6 million objects. Observations on three photometric bands (JHK). The positions accuracy is about 70 mas. No proper motions.

Astrometry and your images

Few people want to deal with the details of positional astronomy, but many people want to know the precise position of objects they have observed.

There are many software tools to aid you in determining an astrometric solution for your data (IRAF, WCS tools, routines in IDL).

But first we need discuss the world coordinate system and FITS images.

World Coordinate System and FITS

In the late 1970's astronomers developed, the Flexible Image Transport System, **FITS**, as an archive and interchange format for astronomical data files.

In the past decade FITS has also become the standard formats for on-line data that can be directly read and written by data analysis software.

FITS is much more than just an image format (such as JPG or GIF) and is primary designed to store scientific data sets consisting of multidimensional arrays and 2-dimensional tables containing rows and columns of data.

A FITS File

A FITS file consists of one or more Header + Data Units (HDUs), where the first HDU is called the "Primary Array".

- The primary array contains an N-dimensional array of pixels. This array can be a 1-D spectrum, a 2-D image or a 3-D data cube.
- Any number of additional HDUs, called "extensions", may follow the primary array.
- Every HDU consists of a ACSII formatted "Header Unit" followed by an optional "Data Unit". Each header unit consists of any number of 80-character records which have the general form :
KEYNAME = value / comment string

The keyword names may be up to 8 characters long and can only contain upper-case letters, the digits 0-9, the hyphen, and the underscore character. The value of the keyword may be an integer, a floating point number, a character string, or a Boolean value (the letter T or F). There are many rules governing the exact format of keyword records so it is usually best to rely on a standard interface software like CFITSIO, IRAF or the IDL astro library to correctly construct or parse the keyword records rather than directly reading or writing the raw FITS file.

FITS/WCS References

For a complete description of the FITS/WCS projections and definitions see :

I: "Representations of world coordinates in FITS", Greisen, E.W. & Calabretta, M.R. (2002),
Astronomy & Astrophysics, 395, 1061-1075.

II: "Representations of celestial coordinates in FITS", Calabretta, M.R., & Greisen, E.W., (2002),
Astronomy & Astrophysics, 395, 1077-1122.

III: "Representations of spectral coordinates in FITS", Greisen, E.W., Calabretta, M.R. Valdes, F.G., & Allen, S.L., (2006), Astronomy & Astrophysics, 446, 747-771.

World Coordinate System

There are a set of FITS conventions that have been defined to specify the physical, or world, coordinates to be attached to each pixel of an N-dimensional image. By world coordinates, one means coordinates that serve to locate a measurement in some multi-dimensional parameter space.

One common example is to link each pixel in an astronomical image to a specific direction on the sky (such as right ascension and declination).

In general the FITS world coordinate system (WCS) of an image is defined by keywords in the FITS header. The basic idea is that each axis of the image has a the following keywords associated with it:

- **CTYPE i** Type of coordinate on Axis i , 8 characters(The FITS WCS standard defined 25 different projections which are specified by the CTYPE keyword.)
- **CRPIX i** Reference pixel on Axis i
- **CRVAL i** Value of World Coordinate at Axis i at reference point (CRPIX i)
- **Cd i_j** A Matrix of partial derivatives of the World Coordinates with respect to the pixel coordinates.

Old style:

- **CDEL T_i** coordinate increment on Axis i
- **CROTA i** rotation parameter for each Axis i

$$\begin{aligned} \text{cd1_1} &= \text{crpix1} \cos(\text{crota2}) \\ \text{cd1_2} &= \text{crpix2} \cos(\text{crota2}) \\ \text{cd2_1} &= -\text{crpix1} \sin(\text{crota2}) \\ \text{cd2_2} &= \text{crpix2} \sin(\text{crota2}) \end{aligned}$$

FITS Header WCS Example for IRAC Mosaicked image

```
SIMPLE      =          T / Fits standard
BITPIX     =         -64 / Bits per pixel
NAXIS      =          2 / Number of axes
NAXIS1     =       13600 / Axis length
NAXIS2     =       16700 / Axis length
EXTEND     =          T / File may contain extensions
ORIGIN     = 'NOAO-IRAF FITS Image Kernel July 2003' / FITS file originator
DATE       = '2008-10-20T16:11:11' / Date FITS file was generated
IRAF-TLM= '13:21:02 (22/10/2008)' / Time of last modification
COMMENT    FITS (Flexible Image Transport System) format is defined in 'Astronomy
COMMENT    and Astrophysics', volume 376, page 359; bibcode: 2001A&A...376..359H
CTYPE1     = 'RA---TAN'
CTYPE2     = 'DEC--TAN'
CRVAL1     =         217.812837962
CRVAL2     =         34.051007516
CDELT1     =         -0.000239631
CDELT2     =         0.000239631
CRPIX1     =          7105.
CRPIX2     =         8073.5
CROTA2     =          0.0
PROGID     =         40839
BUNIT      = 'MJy/sr '
ZEROPT     =          17.997
HISTORY    = 'BCD 16.1.0,17.0.4'
HISTORY    = 'IRACproc-4.1.2; Schuster et al 2006, SPIE, 6270, 65'
HISTORY    = 'Montage-v3.0'
HISTORY    = 'Processed for the Spitzer Deep-Wide Field Survey team'
HISTORY    = 'SDWFS, at the Harvard-Smithsonian Center for Astrophysics'
HISTORY    = 'by MLN Ashby, 2008 October 22. Version 3.2'
END
```

$$\xi = \text{cdelt1} * (x - \text{crpix1}) * \cos(\text{crota2}) - \text{cdelt2} * (y - \text{crpix2}) * \sin(\text{crota2})$$

$$\eta = \text{cdelt1} * (x - \text{crpix1}) * \sin(\text{crota2}) + \text{cdelt2} * (y - \text{crpix2}) * \cos(\text{crota2})$$

Or

$$\xi = \text{cd1_1} * (x - \text{crpix1}) + \text{cd1_2} * (y - \text{crpix2})$$

$$\eta = \text{cd2_1} * (x - \text{crpix1}) + \text{cd2_2} * (y - \text{crpix2})$$

Then

$$\alpha = \tan \text{ projection } (\xi, \eta)$$

$$\delta = \tan \text{ projection } (\xi, \eta)$$

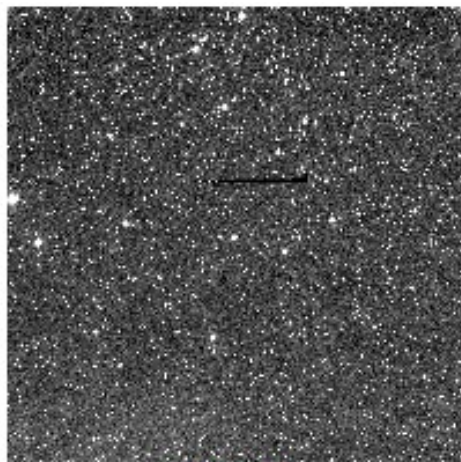
Practical Usage of the FITS/WCS- matching your data to a reference catalog

There are a number of software packages that aid the astronomer in reading, using, or modifying the astrometric information found WCS parameters of the image header. A few of the most commonly used packages are IRAF, WCStools, WCSLIB, and packages in the astronomy IDL library.

If an adequate WCS does not exist for an image the basic steps are:

1. Read in the FITS image and its header
2. Read in a reference catalog (selecting appropriate parameters)
3. Match the reference stars to the image stars – producing a matched reference catalog, pixel values, α, δ
4. Using one of the above WCS software packages perform a fit between the matched star's pixel and α, δ . (The fit can account for various types of distortions on the image, coma, differential astronomical refraction, and telescope flexure). Write the resulting WCS information to the header.
5. Check the result and perform steps 3 and 4 until satisfied

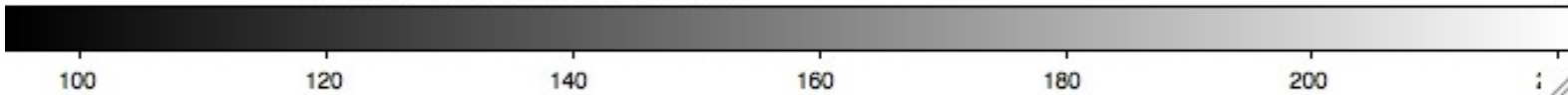
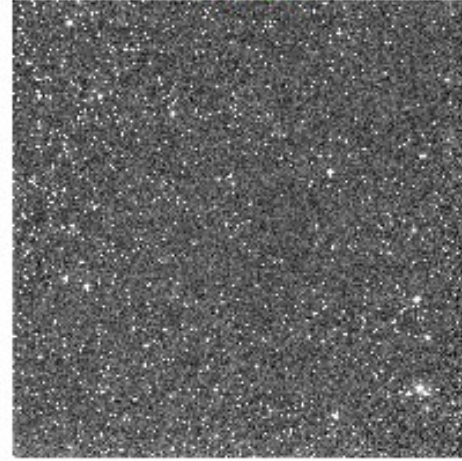
Example of matching a reference catalog to 90-Prime Data



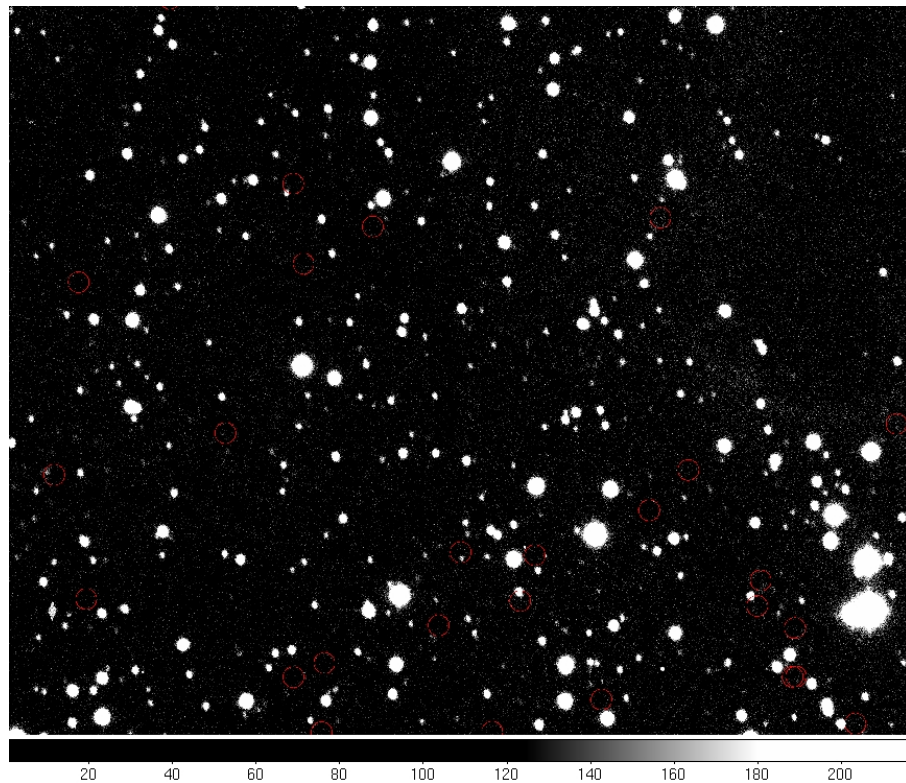
Chip3
Chip1



Chip4
Chip2

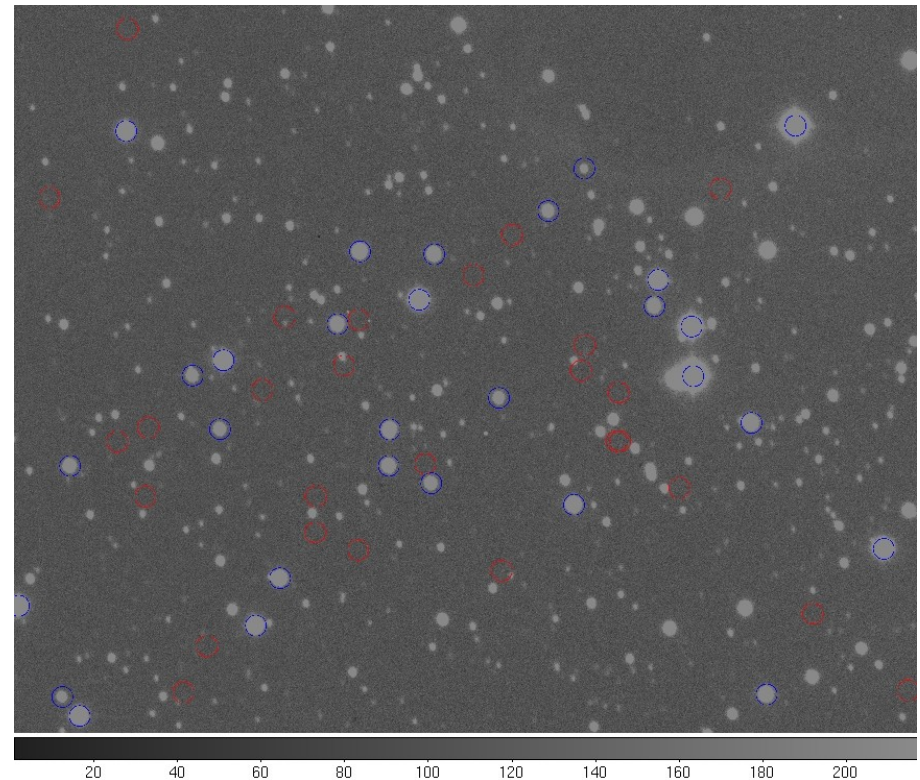


Modifying the WCS information using IRAF and a reference catalog



Misalignment

Red circles – location of reference stars (USNO-A) using crude WCS.



Good alignment

Blue circles- location of USNO A reference stars after updating WCS using MSCFINDER and MSCTPEAK routines from IRAF

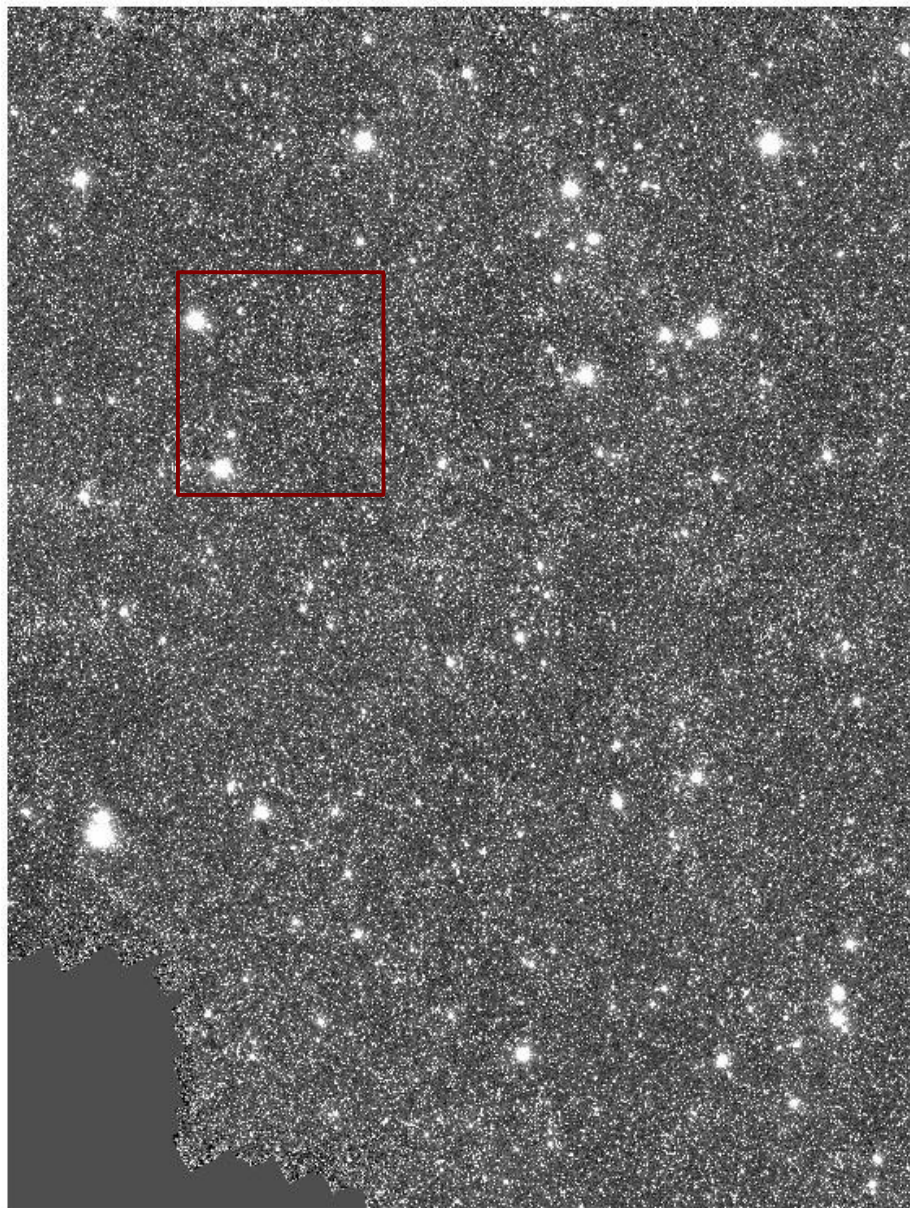
Updated WCS in header for 90-Prime Image- Chip 1

```
CTYPE1 = 'RA---TNX'      /
CTYPE2 = 'DEC--TNX'      / ← Tangent projection + non-linear distortion terms
RADECSYS= 'FK5'         /
EQUINOX =                2000. /
WCSDIM =                 2 /
CRVAL1 = 39.3873750000 /
CRVAL2 = 62.058361111111 /
CRPIX1 = 4603.78554581209 /
CRPIX2 = 4640.70823246779 /
CD1_1 = 4.87540234153667E-7 /
CD1_2 = 1.25515025162259E-4 /
CD2_1 = 1.25579483259406E-4 /
CD2_2 = -1.1304526748985E-6 /
LTM1_1 = 1.00000 /
LTM2_2 = 1.00000 /
WAT0_001= 'system=image' /
WAT1_001= 'wtype=tnx axtype=ra lngcor = "3. 5. 5. 2. -0.5845971386865003 -0.068'
WAT2_001= 'wtype=tnx axtype=dec latcor = "3. 5. 5. 2. -0.5845971386865003 -0.06'
OBSERVAT= 'OBSERVAT' /
WAT1_002= '61663338818925 -0.577399663518512 -0.05852247586753612 0.00399833267'
WAT1_003= '9100997 0.08101206082055306 0.4527117805067316 1.009355945605757 0.8'
WAT1_004= '363757921265274 -0.01576740706164705 -0.04625344566246046 0.07192274'
WAT1_005= '909887355 -0.01300890280834304 -9.212214856578248E-4 -0.150422717957'
WAT1_006= '2659 0.1920717664694512 0.1734947211726993 -0.1946868528237631 0.262'
WAT1_007= '9623022570727 "'
WAT2_002= '861663338818925 -0.577399663518512 -0.05852247586753612 0.0103030337'
WAT2_003= '8749223 0.03178326805266181 -0.06189414518588323 -0.518029769559462 '
WAT2_004= '-0.6476076457433045 0.1210466639957179 0.5420860985636986 1.54020743'
WAT2_005= '894359 1.376263888565729 0.3745320908303613 0.3233457867507124 0.558'
WAT2_006= '9147886417743 0.767458255327182 -0.0603653590331878 0.68256448673508'
WAT2_007= '95 " '

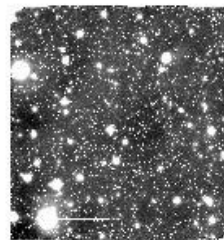
```

Distortion coefficients:
WAT_i_nnn

Astrometry example: Matching sources in two images from different instruments



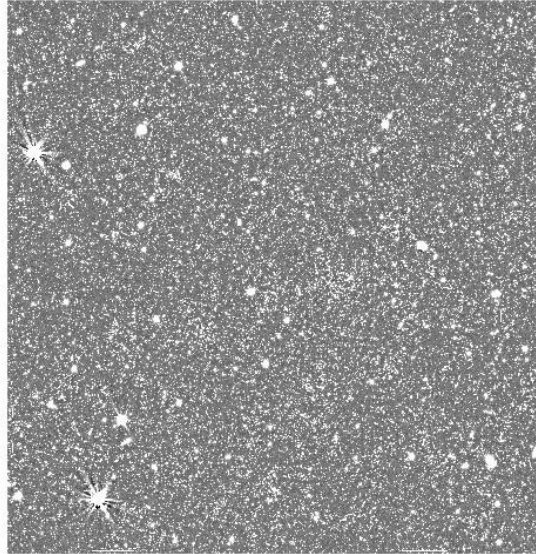
Pixel sizes are different
and images sizes are
different



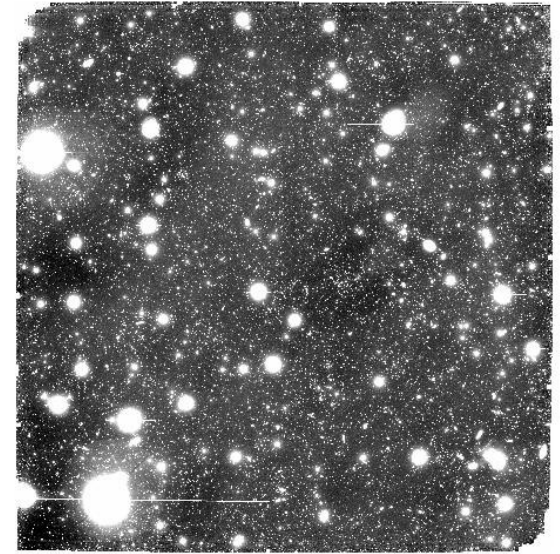
NOAO I band image

IRAC Mosaic image

IRAC Image
cropped and WCS modified



NOAO I-band Image



Zoomed region
After running:

- Wregister
(use polynomial order 3)
- SExtractor
- Scamp

Match objects Circled

