Nearest Point on a Great Circle

Date: 05/27/2002 at 06:19:57
From: Jayasundara
Subject: projection of a point to a great circle

I am studying spherical geometry, and need help with the following question.

Given points A, B, and C on the surface of a unit sphere, I need to find a point P on the great circle arc defined by A and B such that the distance from C to P is equal to the perpendicular distance.

I found a solution to calculate the perpendicular distance from point C to the great circle defined by points A and B. Then, I think if I have the equation of the great circle, I could find the point on it which gives same perpendidular distance from the given point C.

But, I have no clear idea for this. Could you please comment on this.

Thanks,
Jayasundara

Date: 05/27/2002 at 11:36:02
From: Doctor Rick
Subject: Re: projection of a point to a great circle

Hi, Jayasundara.

I don't know what sort of spherical geometry you are learning. I find that the easiest way to work with points on spheres is to use vector algebra. Thus a point on the sphere is represented by any vector from the center of the sphere that passes through the given point.

A great circle is uniquely identified by a vector perpendicular to the plane of the great circle. Given two points on the great circle (your A and B), you can easily find such a vector G by taking the cross product (or vector product) of vectors corresponding to the points.

It's also easy to find the great circle passing through a point C and perpendicular to the great circle specified by vector G. You're

looking for a vector F perpendicular to this great circle, therefore perpendicular to vector C, and also perpendicular to the vector G.

Once you have the vectors F and G, it isn't hard to find the points of intersection of the two great circles. How will these points be related to vectors F and G?

If you're not using vector algebra, I can't offer a whole lot of help.

- Doctor Rick, The Math Forum
  http://mathforum.org/dr.math/

Date: 05/28/2002 at 04:05:25
From: Jayasundara
Subject: projection of a point to a great circle

Thank you very much for the reply.

I hope that to get the perpendicular vector to the plane of the great circle, we have to normalize the vector G (G = A x B). Is that correct?

Now I have vector G perpendicular to A and B (also perpendicular to the plane of the great circle).

I also have vector F = (C x G) / ||C xG||, which is perpendicular to C and G (also perpendiculer to the plane of the great circle).

Then taking the cross product of F and G and normalizing the resultant vector, I have the intersection points of the great circles (+/- (F x G) / ||F xG||). These are antipodal points.

I hope in this way these points will be related to vectors F and G.

Is my argument is correct? Please let me know.

Thanks a lot,
Jayasundara

Date: 05/28/2002 at 08:39:36
From: Doctor Rick
Subject: Re: projection of a point to a great circle

Hi, Jayasundara.

For your purposes, normalization is not necessary until the last step, when you convert a vector into coordinates of a point on the sphere. If we needed to find an angle between vectors (other than 90 degrees) using the dot (scalar) product of two vectors, the magnitudes of the vectors would matter, and normalization is appropriate; but your problem does not require this. It doesn't hurt, though, so if you want all vectors to be unit vectors, go ahead and normalize at each step.

Vector G = A x B is perpendicular to the plane of the great circle. The vector F = C x G is perpendicular to C, so the great circle it defines passes through C. It is also perpendicular to G, so the great circle it defines is perpendicular to the great circle defined by G. The intersections of the two great circles correspond to vectors that are on both great circles, and thus perpendicular to both F and G.

Your formulation is correct.

- Doctor Rick, The Math Forum
  http://mathforum.org/dr.math/

Date: 05/28/2002 at 09:54:14
From: Jayasundara
Subject: Thank you

Thanks a lot for the information and immediate response.

Jayasundara

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