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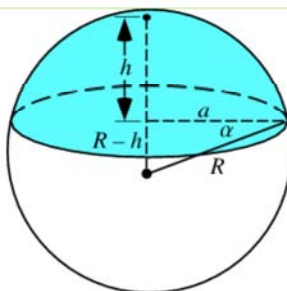
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Spherical Cap



A spherical cap is the region of a sphere which lies above (or below) a given plane. If the plane passes through the center of the sphere, the cap is called a hemisphere, and if the cap is cut by a second plane, the spherical frustum is called a spherical segment. However, Harris and Stocker (1998) use the term "spherical segment" as a synonym for what is here called a spherical cap and "zone" for spherical segment.

Let the sphere have radius R , then the volume of a spherical cap of height h and base radius a is given by the equation of a spherical segment

$$V_{\text{spherical segment}} = \frac{1}{6} \pi h (3a^2 + 3b^2 + h^2) \quad (1)$$

with $b = 0$, giving

$$V_{\text{cap}} = \frac{1}{6} \pi h (3a^2 + h^2). \quad (2)$$

Using the Pythagorean theorem gives

$$(R-h)^2 + a^2 = R^2, \quad (3)$$

which can be solved for a^2 as

$$a^2 = 2Rh - h^2, \quad (4)$$

so the radius of the base circle is

$$a = \sqrt{h(2R-h)}, \quad (5)$$

and plugging this in gives the equivalent formula

$$V_{\text{cap}} = \frac{1}{3} \pi h^2 (3R-h). \quad (6)$$

In terms of the so-called contact angle (the angle between the normal to the sphere at the bottom of the cap and the base plane)

$$\begin{aligned} R-h &= R \sin \alpha \\ \alpha &= \sin^{-1} \left(\frac{R-h}{R} \right), \end{aligned} \quad (7)$$

so

$$V_{\text{cap}} = \frac{1}{3} \pi R^3 (2 - 3 \sin \alpha + \sin^3 \alpha). \quad (9)$$

The geometric centroid occurs at a distance

$$\bar{z} = \frac{3(2R-h)^2}{4(3R-h)} \quad (10)$$

above the center of the sphere (Harris and Stocker 1998, p. 107).

The cap height h at which the spherical cap has volume equal to half a hemisphere is given by

$$h_{1/2} = 1 - 2 \cos \left(\frac{4}{9} \pi \right). \quad (11)$$

Consider a cylindrical box enclosing the cap so that the top of the box is tangent to the top of the sphere. Then the enclosing box has volume

$$V_{\text{box}} = \pi a^2 h \quad (12)$$

$$= \pi (R \cos \alpha)^2 [R(1 - \sin \alpha)] \quad (13)$$

$$= \pi R^3 (1 - \sin \alpha - \sin^2 \alpha + \sin^3 \alpha), \quad (14)$$

so the hollow volume between the cap and box is given by

$$V_{\text{box}} - V_{\text{cap}} = \frac{1}{3} \pi R^3 (1 - 3 \sin^2 \alpha + 2 \sin^3 \alpha). \quad (15)$$

The surface area of the spherical cap is given by the same equation as for a general zone:

$$S_{\text{cap}} = 2\pi R h \quad (16)$$

$$= \pi (a^2 + h^2). \quad (17)$$

SEE ALSO:

Contact Angle, Dome, Frustum, Hemisphere, Solid of Revolution, Sphere, Spherical Ring, Spherical Segment, Spherical Wedge, Surface of Revolution, Torispherical Dome, Zone



computational knowledge engine

- THINGS TO TRY:
- spherical cap
 - $2 * 4 * 6 * \dots * 36$
 - cubic crystal family

REFERENCES:

Harris, J. W. and Stocker, H. "Spherical Segment (Spherical Cap)." §4.8.4 in *Handbook of Mathematics and Computational Science*. New York: Springer-Verlag, p. 107, 1998.

Kern, W. F. and Bland, J. R. "Spherical Segment." §36 in *Solid Mensuration with Proofs, 2nd ed.* New York: Wiley, pp. 97-102, 1948.

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