Array Phasing via the Time Delay Method

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June, 2012

I desired to modify the SuperJove array, in part by removing the extra gap from the middle of the array, in so doing making all four elements evenly spaced at 20' apart. I am calling the new array the SuperJove Version 2. See *The JOVE Bulletin*, Nov 2011, **http://radiojove.gsfc.nasa.gov/library/newsletters/toc.htm** for a description of the original array.

This change to the element spacing will necessitate changes to the array's delay cable lengths. This article describes the procedures used to steer the beam of a phased array using the time delay method. The method of steering the beam of an antenna array works with any type of array element; for example, dipoles, Yagis, log periodics, and TP antennas (conical log spirals).

Delay Cable Calculations

We must first know what we want to look at and where we are so we know where to aim the beam. I primarily want to observe Jovian emissions. The array is at 29.5°N latitude and Jupiter at transit is at 20°N declination (a good approximate average through the end of 2014). The desired beam elevation can be found by first finding the elevation of the celestial equator, then adding the declination of Jupiter.

$$\text{Desired Beam Elevation} = \left(90° - 29.5°\right) + 20° \cong 80°$$

A diagram of the layout of the array helps envision how delay cables are used to steer the beam.

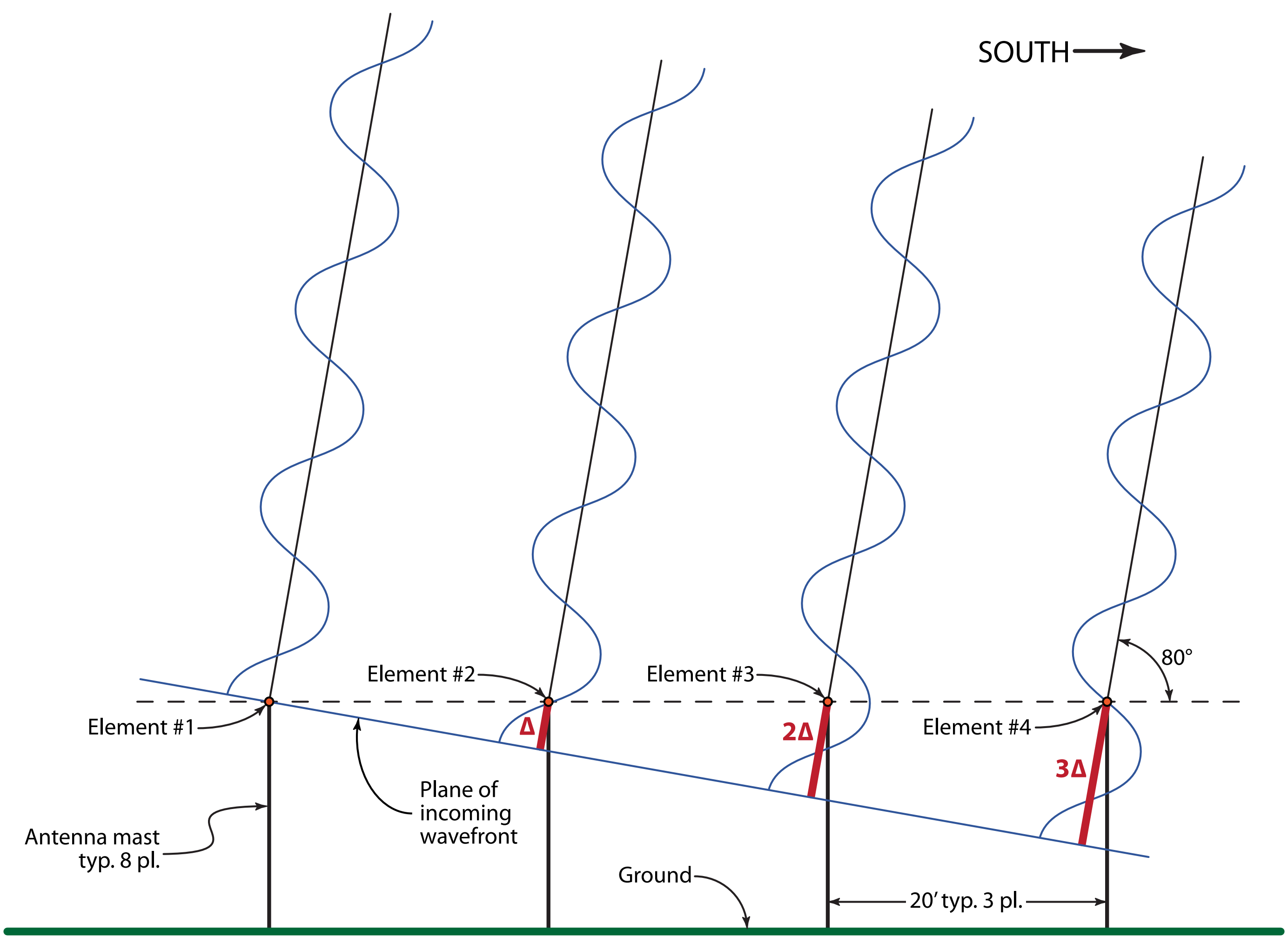

Figure 1 – SuperJove Version 2 array, looking East toward the ends of the element wires.

If we envision plane waves coming toward the array from 80° elevation to the south, it can be seen that the wave plane will strike the southern-most element first and the northern-most element last. If we send the signals from the elements directly to a power combiner without taking this fact into account, the signals will combine out of phase and thus the array's response won't be at its highest. That's another way of saying that the array's beam will not be pointed at 80° elevation, but somewhere else (with equal feed line lengths between each element and a power combiner, the beam is directed toward zenith, or 90° elevation).

To aim the beam of the antenna array toward 80° elevation, we must add a delay into each of the three feed lines connected to the three southern elements (elements 2 through 4). This can be done easily by adding the appropriate lengths of coax cable to each element's feed line. The northern element needs no delay because it is the last to receive a signal from 80° elevation. With the right amount of delay in each feed line, all four signals reach the combiner at the proper time and are therefore combined in phase, which is another way of saying that the beam is aimed south with an 80° elevation.

To calculate the proper delay for each array element, we must first calculate the extra distance the plane wave must travel to reach one element after the next. Since the SuperJove Version 2 array has equally spaced elements (unlike the original SuperJove array), all the delays will be multiples of the same incremental delay.

The equation for each incremental increase (Δ) in delay distance (free space travel distance) is:

$$\text{Free Space Delay Distance } \Delta = \left(\text{Inter-Element Horizontal Distance}\right)\cos\left(\text{Beam Elevation}\right) \tag{1}$$

For the SuperJove Version 2 array, we have:

$$\boxed{\text{Free Space Delay Distance } \Delta = \left(20'\right)\cos 80^\circ = 3.47'}$$

So, what we need is a way to add multiples of 3.47 feet of extra free space travel distance to the signals before they reach the power combiner. That is, the signals need to be delayed by the extra length of travel distance. The figure calculated above, however, is the distance in free space. We must convert this to the distance needed when using coax. Since RF moves slower in coax than it does in free space, the length will be foreshortened by the degree to which the signal is slowed down.

For the SuperJove Version 2 array, I am using RG-58 (Belden 8259) and Times Microwave LMR-400 coax. The velocity of propagation (VoP) for any coax cable may be found on the manufacturer's data sheet, or may be measured with an antenna analyzer or vector network analyzer. Belden 8259 has a VoP of 66% and LMR-400 has a VoP of 85%. This represents the signal velocity as a fraction of the speed of light in free space.

The equation for the equivalent incremental delay cable length in a coax is:

$$\text{Delay Cable Length} = \left(\text{Free Space Delay Distance } \Delta\right)\left(\text{VoP}\right) \tag{2}$$

The incremental lengths of the delay cables for the SuperJove Version 2 array are thus:

$$\boxed{\begin{aligned}\text{Incremental Delay Cable Length (Belden 8259)} &= \left(20'\right)\cos 80^\circ\left(0.66\right) = 2.29' = 2'3\tfrac{1}{2}''\\ \text{Incremental Delay Cable Length (LMR-400)} &= \left(20'\right)\cos 80^\circ\left(0.85\right) = 2.95' = 2'11\tfrac{1}{2}''\end{aligned}}$$

Note that these delay cables are most often called *time delay* cables because they add extra travel time to the signal path. Extra time or extra distance, it all works out the same as long as you do the math right. While we don't need to know the actual delay *time*, here is the equation for the incremental delay time, just for gee whiz purposes:

$$\text{Incremental Delay Time} = \frac{\text{Free Space Delay Distance } \Delta}{\text{Speed of Light}} = \frac{\left[20\,\text{ft}\left(\dfrac{1\,\text{m}}{3.28\,\text{ft}}\right)\right]\cos\left(80^\circ\right)}{3.00\times 10^{8}\ \text{m/s}} = 3.53\,\text{ns}$$

A delay of 3.53 ns checks well against the 3.47 feet calculated above, remembering that in free space (*not in coax cable*), the speed of light is just shy of one foot per nanosecond.

It often makes the remainder of the array feed system design work go easier if we convert the incremental free space delay distance into a phase in degrees at a certain frequency. It doesn't really matter what frequency you pick, since this will be converted back to cable lengths using the same frequency. It is common to use the design center frequency of the antenna (in this case, 20.1 MHz), just for good form if nothing else. The equation for the equivalent incremental phase delay (ϕ) at a given frequency f is:

$$\text{Incremental Phase Delay}\,\phi = 360^\circ \frac{(\text{Free Space Delay Distance }\Delta)\,f}{\text{Speed of Light}} \tag{3}$$

For the incremental phase delay at 20.1 MHz, we have:

$$\boxed{\text{Incremental Phase Delay }\phi = 360^\circ \frac{\left[20\,\text{ft}\left(\frac{1\,\text{m}}{3.28\,\text{ft}}\right)\right]\cos(80^\circ)\left(2.01\times10^{7}\,/\text{s}\right)}{3.00\times10^{8}\ \text{m/s}} \cong 26^\circ}$$

As long as it is remembered that the phase delay ϕ is in this case 26° only at 20.1 MHz, then we can use this phase delay to generate the required lengths of the delay cables. This comes in very handy when calculating and verifying the physical layout and lengths of all the feed lines, for working with free space degrees is much easier than horsing around with feet and inches and VoP.

Calculation of Feed Line Lengths

Knowing the incremental time delay in terms of a phase at some frequency, we can plan the layout of the feed lines. Again, a diagram helps make everything a little clearer.

Looking at Figures 1 and 2, we see that we must insert 3Δ of delay on the southern-most element (#4), 2Δ of delay on element #3, 1Δ of delay on element #2, and no delay at all on the northern-most element (#1). This arrangement of delays will allow the signals from 80° elevation to be combined in phase.

One way to do this is with a 4-way combiner and four different feed line lengths, one for each element. The down side to that approach is that you end up with higher losses because the feed lines have to be long enough to actually reach the power combiner. Dipole feed lines are usually lightweight—LMR-240 or RG-58, for example—and have higher losses than heavier, larger diameter coax—LMR-400 or RG-8, for example, which have lower losses.

The way I chose to lay out the feed system is with three 2-way power combiners, keeping the small diameter (higher loss) coax shorter, and using larger diameter,

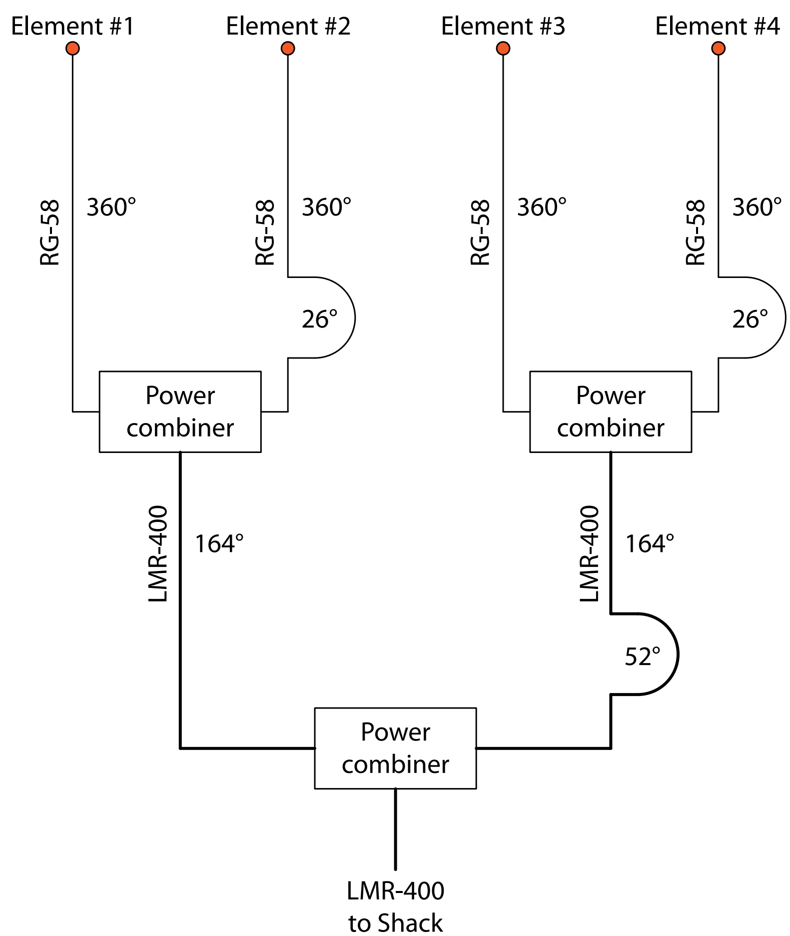

Figure 2 – Feed line layout of the SuperJove Version 2 array.

low-loss coax to combine the two halves of the array, all while maintaining the proper delay relationships between the elements. It's really not that difficult to do this.

We start by finding the minimum coax length needed for the northern-most element; this will be the shortest feed line of the four. I chose 1λ at 20.1 MHz, or 32'4" because it fits well with the physical dimensions of the array; you don't want the feed line so short that you can't move it around to cut the grass (see Equation 4 below and use $\phi = 360°$).

For this layout, we work one pair of elements at a time. The southern elements in each half of the array (element #2 and element #4) will have 1Δ of delay—and thus 1ϕ of phase delay—added to its 32'4" feed line.

Then, we must combine the two array pairs. To find the delay needed for that, we simply note from Figure 1 that the southern pair of dipoles is 2Δ away from the northern pair (in terms of delay distance). So, the LMR-400 feed from the southern pair's combiner to the final combiner needs to have 2Δ more signal path in free space (and consequently 2ϕ—or 52°—of phase delay at 20.1 MHz) compared to the LMR-400 feed from the northern pair's combiner.

To convert phase delay ϕ in degrees to length in coax at frequency *f*, we use:

$$\text{Coax Length} = \frac{\phi}{360°}\left(\frac{\text{Speed of Light}}{f}\right)(\text{VoP}) \tag{4}$$

For the 52° in LMR-400, we have:

$$\text{Coax Length} = \frac{52°}{360°}\left(\frac{3.00\times10^{8}\ \text{m/s}}{2.01\times10^{7}\ \text{/s}}\right)(0.85)\left(\frac{3.28\,\text{ft}}{1\,\text{m}}\right) = 6.01\,\text{ft} = 6'0''$$

It is often good practice when steering the beam of narrow band antennas to keep the lengths of the non-phased cables to integer multiples of a half wavelength at the design frequency when possible so the cable does not act as an impedance transformer. However, I did not want to use a full 360° (1λ) or 180° (½λ) of LMR-400. For one thing, the element impedance in the SuperJove Array is very close to 50 ohms, so any given length of coax will have only a small effect on the apparent impedance of the elements presented to the power combiners. For another thing, LMR-400 is expensive—and since the VoP is high, I didn't want yards of the stuff laying all over the place unnecessarily. (As luck would have it, the non-phased portion of the LMR-400 turned out pretty close to 180° anyway at 20.1 MHz. *Good enough.)*

I looked at the distances on the ground that I needed to cover and decided I wanted a total of 44' of cable between the two first stage power combiners. With that, we can calculate the lengths of the two LMR-400 feed lines via 2x + 6 ft = 44 ft → x = 19 ft, therefore the northern section of LMR-400 needs to be 19' and the southern section needs to be 6' longer at 25'.

Now that we have the actual lengths of the LMR-400 sections, we just have to convert those back to phase delays so we can double check everything (see below). We use the inverse of equation 4:

$$\text{Phase Delay } \phi = 360°\ \frac{(\text{Coax Length})\,f}{\text{Speed of Light }(\text{VoP})} \tag{5}$$

For 19 ft of LMR-400 at 20.1 MHz, we have:

$$\text{Phase Delay } \phi = 360°\ \frac{\left[19\,\text{ft}\left(\frac{1\,\text{m}}{3.28\,\text{ft}}\right)\right](2.01\times10^{7}\ \text{/s})}{3.00\times10^{8}\ \text{m/s}\,(0.85)} \cong 164°$$

Similarly, we find that 25' of LMR-400 at 20.1 MHz is equal to 216°, which is 52° longer than the 19' cable, just as it should be.

Verification of Calculations

The last—*and most important*—step is to verify all this fancy math. This is done by making a table, one column for each element, and adding up all the phase delays between each element and the final power combiner. Trace the signal path from each element to the final power combiner, noting the length of the signal path in degrees along the way.

Add these up to find the total feed line path length in degrees for each element in the array. Then, normalize all the path lengths to element #1 by subtracting element #1's path length from all four sums; this will show each element's delay in degrees relative to element #1.

If the feed system was calculated properly, all the delays should be integer multiples of ϕ in an arrangement that agrees with Figure 1. Once this has been verified, then the phase delays can be converted to lengths in the appropriate type of coax cable using Equation 4 (see Table 2).

	Element 1	**Element 2**	**Element 3**	**Element 4**
Desired ϕ Delay	**0°**	**26°**	**52°**	**78°**
RG-58	360°	360°	360°	360°
RG-58 Delay	**0°**	26°	**0°**	26°
LMR-400	164°	164°	164°	164°
LMR-400 Delay	**0°**	**0°**	52°	52°
Total Line Length	524°	550°	576°	602°
Normalize to Element #1	– 524°	– 524°	– 524°	– 524°
ϕ Delay Relative to Element #1	0° ✓	26° ✓	52° ✓	78° ✓

Table 1 – Design verification of the feed system using phase delays at 20.1 MHz.

RG-58 (Belden 8259) VoP = 66%	**Times LMR-400** VoP = 85%
360° = 32′4″	164° = 19′
26° = 2'4"	216° = 25′

Table 2 – Phase delays at 20.1 MHz converted to the required delay cable lengths (see Equation 4).

Summary

Phasing an array—that is, steering the array's beam to a desired spot in the sky—is not that difficult once the time delay method is understood. At the risk of introducing confusion, it is worth mentioning that you will run into discussions about beam steering using phase angle alone. That sort of analysis works well *at one frequency*—but it does *not* work for broadband antennas like log periodics or conical log spirals. A broadband antenna array requires the time delay method.

It should be noted that the phase angle method of phasing an array is simply a special case of the time delay method, where the assumption is made that the antenna operates only at one frequency. This is useful and appropriate for narrow band antennas like dipoles and Yagis (the time delay method works fine for narrow band elements; it just involves more calculation than the phase angle method).

Regardless of the method you use, it pays to be careful in your thinking, for a phase delay at one frequency is not the same phase delay at another frequency—but a time delay is the same at all frequencies.

For the time delay method, you calculate the required aim point, calculate the length of the delay cables, and verify the calculations. The only thing left is to model the array in EZNEC or NEC-Win, build the array, and test it under real world conditions. These latter three steps will be the subject of another article.